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Sensitivity of the numerical solution to finite element mesh for reinforced concrete deep beams[1]

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An analysis of the influence of the manner of dividing the structure on the numerical solution of the static problems of the concrete and of the reinforced concrete deep beams, using a constitutive model of the concrete that demonstrates the material softening, is given. Detailed results of the numerical solutions are presented in the paper. The results indicate that taking into account the scale parameters makes it possible to increase the objectivity of the numerical results of modelling of the behaviour of concrete and reinforced concrete structures when the material softening is considered. The numerical analysis for the reinforced concrete deep beams indicates the differentiation of the obtained results according to the fracture energy values.

1. Introduction

It is observed in the literature that various material models are applied to the description of the nonlinear behaviour of concrete in the solution within the mechanics of the reinforced concrete structure domain, e.g. [5]. A lot of authors underline in theirs papers that the application of complex models, including the material softening effect, to the description the concrete properties leads to the generation of additional problems connected with the application of numerical methods to the analysis of discretised models of the structure, e.g. [1, 7, 8, 15]. One of such problems is the sensitivity of the numerical solution to the manner of the structure discretisation. Different numerical methods that lead to results insensitive to the finite element mesh were discussed by Łodygowski [11, 12]. Łodygowski stated that the application of the regularisation method using viscoplasticity guarantees uniqueness and stability of the initial boundary value problem and allows to set free of the effects resulting from the sensitivity to the finite element mesh. On the other hand, de Borst has indicated [1] an effective method of modelling the structure behaviour by using the so-called scale effect of the finite element mesh.

This paper is aimed at the analysis of the sensitivity of numerical solution of the static problem of concrete and reinforced concrete deep beams to the manner of dividing the structure into the finite elements, when a model of the concrete that demonstrates the material softening, is used. Moreover, with reference to the reinforced concrete deep beams, the problem of the influence of the scale parameters not only on the process of the free the numerical solution from the manner of the structure discretisation, but also on the numerically simulated fracture mechanism in comparison to the mechanism observed in the experiment [9], is discussed.

The reinforced concrete deep beam is treated as a material composition which consists of a concrete matrix reinforced by slender steel bars distributed in the matrix material. Modelling of the structural material properties was achieved using the plastic flow theory assumptions. An elastic-perfectly plastic material model was applied for the reinforcement steel. A reduced static form of the non-standard model of dynamic deformation [14], in which the determination of the dynamic

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strength was disregarded, was employed for the concrete. The model describes elastic properties of the concrete, its limited perfectly plastic properties, and the material softening and material dilatation. The degradation of the elastic material constants was disregarded. The unloading processes are linear and elastic. The applied model of concrete enables a simplified description of the cracking or crushing of the material as states of the loss of load-capacity which are reached in the material softening process during tension or compression, respectively. The analysis of the numerical solution sensitivity to the manner of structure discretisation requires the consideration of parameters which describe the so-called scale effect of the finite elements of the discretised structure in the deformation model of concrete. The fracture energy determined experimentally and the equivalent length of the finite element mesh were assumed as the scale parameters in the present paper.

The method of the structure system effort analysis was developed using the finite element method [6, 16]. The hypothesis of reinforcement bars and the matrix material joint-action was proposed for the reinforced concrete deep beams. The system of the finite element method incremental equilibrium equations was solved on the basis of the algorithms and procedures of numerical analysis of the plane stress state problem. This enables the determination of the displacement, strain, and stress states with regard to physically nonlinearities of the material, including cracking and crushing processes in the concrete.

2. MODELLING OF THE STRUCTURAL MATERIALS PROPERTIES

2.1. Model of the reinforcement steel

Considering the function of the deep beam's reinforcement as slender steel bars, the elastic–perfectly plastic deformation model in the uniaxial stress state was applied to the reinforcement steel.

The following equations describe the reinforcement steel model,

$$\sigma_a = \left\{ \begin{array}{l} E_a \varepsilon_a \\ \pm R_a \end{array} \right. , \qquad \Delta\sigma_a = \left\{ \begin{array}{l} E_a \Delta\varepsilon_a \\ 0 \end{array} \right. , \qquad \text{for} \quad |\varepsilon_a| \left\{ \begin{array}{l} \leq \varepsilon_e \\ > \varepsilon_e \end{array} \right. , \quad \varepsilon_e = \frac{R_a}{E_a} , \tag{1}$$

where: E_a – modulus of elasticity; R_a – tensile–compressive strength of steel.

2.2. Model of the concrete

2.2.1. Basic version of the model

Reduced, form of the non-standard model of dynamic deformation was applied to the concrete [14]. The reduction is based on the disregarding of the determination of the dynamic strength of the concrete and makes it possible to describe the static elastic–plastic behaviour of the material taking into account its softening. The basic elements of model are given below.

The equation of the limit surface, depending on three stress invariants, is assumed in the following form,

$$F(\sigma_{ij}) = \left[\frac{\tau_0}{\rho(\varphi)} + A \right]^2 - B\sigma_0 - C = 0 \tag{2}$$

where: τ_0 – tangent octahedral stress as function of the second invariant of the stress deviator; σ_0 – mean normal stress as function of the first invariant of the stress; $\rho(\varphi)$ – function determining the shape of the limit surface cross-section at the octahedral plane σ_0 = const, depending on the third invariant of the stress deviator; A, B, C – material constants which are functions of the basic strength of the concrete for the uniaxial and biaxial compression and the uniaxial tension.

The properties of the limit surface, especially its good agreement with the experimental results for the concrete in the complex stress states, are the reason for applying equation (2) as a yield surface for the elastic–plastic material. To this end, the parameter K which graduates the basic strength of the concrete, was also introduced. Thus, the yield surface equation can be written in the following form:

$$F(\sigma_{ij}, K) = \left[\frac{\tau_0}{\rho(\varphi)} + Ka\right]^2 - Kb\sigma_0 - K^2 c = 0. \tag{3}$$

The interpretation of the K parameter as a material softening parameter makes it possible to describe the yield surface evolution.

A three-phase idealisation of the concrete behaviour was assumed. The following deformation phases are distinguished: 1° – elastic attaining of the initial yield surface; 2° – perfect plastic flow in the limited range of deformation; 3° – material softening modelled as a plastic flow on the transient yield surface whose isotropic shrinkage process is controlled by the softening parameter K that depends on the effective plastic strain and softening modulus modified depending on the up-to-date stress state.

The interpretation of the softening parameter variation corresponding to the three-phase approximation of the concrete behaviour for an uniaxial compression is shown in Fig. 1.

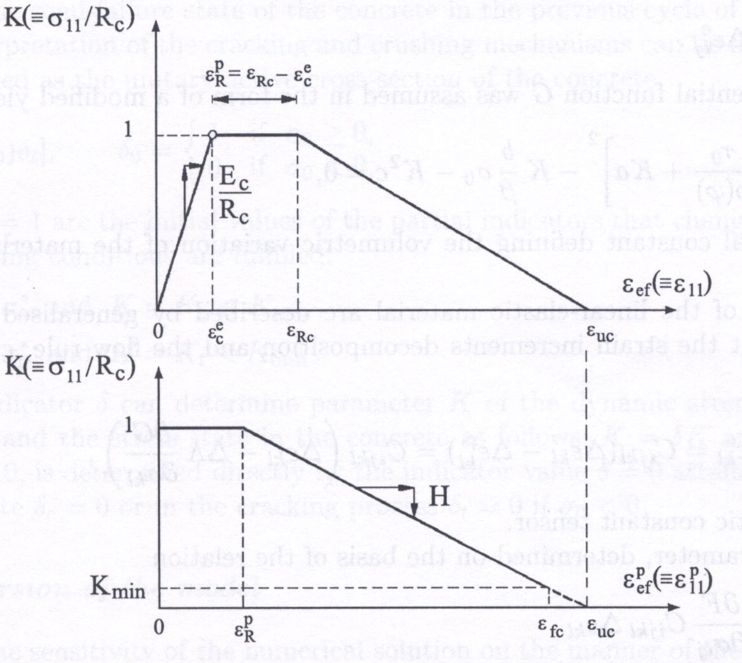

Fig. 1. Interpretation of softening parameter variation

The softening parameter is defined in the following manner:

$$K = \begin{cases} 1 & \text{for} \quad \varepsilon_{\text{ef}}^{\text{p}} \le \varepsilon_R^{\text{p}}, \\ \int_{\varepsilon_R^{\text{p}}}^{\varepsilon_{\text{ef}}^{\text{p}}} \Delta K(\varepsilon_{\text{ef}}^{\text{p}}) & \text{for} \quad K \ge K_{\min} \quad \text{and} \quad \varepsilon_{\text{ef}}^{\text{p}} > \varepsilon_R^{\text{p}}, \\ 0 & \text{for} \quad K < K_{\min} \quad \text{and} \quad \varepsilon_{\text{ef}}^{\text{p}} > \varepsilon_R^{\text{p}}, \end{cases} \tag{4}$$

where: $\varepsilon_{\text{ef}}^{\text{p}}$ – effective plastic strain; ε_R^{p} – limit plastic strain in the perfect plastic flow phase.

The following relation describes the softening parameter variation.

$$\Delta K = \begin{cases} 0 & \text{for} \quad \varepsilon_{\text{ef}}^{\text{p}} \le \varepsilon_R^{\text{p}}, \\ H(\sigma_i^0)\Delta\varepsilon_{\text{ef}}^{\text{p}} & \text{for} \quad \varepsilon_{\text{ef}}^{\text{p}} > \varepsilon_R^{\text{p}}. \end{cases} \tag{5}$$

The nondimensional softening modulus has the form

$$H(\sigma_i^0) = -\frac{1}{\sigma_i^0\left(\varepsilon_{uc} - \varepsilon_{Rc} + \frac{R_c}{E_b}\right)}, \tag{6}$$

where: σ_i^0 – nondimensional stress intensity related to the initial yield surface; ε_{uc}, ε_{Rc} – limit plastic strains; E_c – modulus of elasticity; R_c – uniaxial compressive strength of the concrete.

From the analysis of the experimental results for the uniaxial compression, the basic values of the limited strains ε_{Rc} and ε_{uc}, are taken as constant values independent of strain rate

$$\varepsilon_{Rc} = 0.2\%, \qquad \varepsilon_{uc} = (0.6\text{–}1.2)\%, \tag{7}$$

where lower values of ε_{uc} might be used for the high-class concrete and higher values for the low- and mean-class concrete.

The tensor of the plastic strain increment is defined by postulating the nonassociated flow rule

$$\Delta\varepsilon_{ij}^{\mathrm{p}} = \Delta\Lambda \frac{\partial G}{\partial \sigma_{ij}}, \tag{8}$$

with the assumption of the elastic–plastic strain increments decomposition

$$\Delta\varepsilon_{ij} = \Delta\varepsilon_{ij}^{\mathrm{p}} + \Delta\varepsilon_{ij}^{\mathrm{e}}. \tag{9}$$

The plastic potential function G was assumed in the form of a modified yield surface function

$$G(\sigma_{ij}, K) = \left[\frac{\tau_0}{\rho(\varphi)} + Ka\right]^2 - K\frac{b}{\beta}\sigma_0 - K^2c = 0, \tag{10}$$

where: β – material constant defining the volumetric variation of the material during the plastic deformation.

The properties of the linear-elastic material are described by generalised Hooke's law which, taking into account the strain increments decomposition and the flow rule, can be written in the following form

$$\Delta\sigma_{ij} = C_{ijkl}\,\Delta\varepsilon_{kl}^{\mathrm{e}} = C_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{\mathrm{p}}) = C_{ijkl}\left(\Delta\varepsilon_{kl} - \Delta\Lambda\frac{\partial G}{\partial\sigma_{kl}}\right), \tag{11}$$

where: C_{ijkl} – elastic constant tensor.

The loading parameter, determined on the basis of the relation

$$\Delta\Lambda = \frac{\dfrac{\partial F}{\partial\sigma_{ij}} C_{ijkl}\,\Delta\varepsilon_{kl}}{\dfrac{\partial F}{\partial K} A(H, \eta_{\mathrm{ef}}) + \dfrac{\partial F}{\partial\sigma_{ij}} C_{ijkl}\dfrac{\partial G}{\partial\sigma_{kl}}}, \tag{12}$$

enables the defining of the loading and unloading processes in the following way,

loading:	$\Delta\Lambda > 0$	if	$F = 0$ and $\Delta F = 0$,
unloading:	$\Delta\Lambda = 0$	if	$F < 0$ or $F = 0$ and $\Delta F < 0$.

The applied deformation model of the concrete enables a simplified modelling of the cracking or crushing mechanisms (i.e. the formation, opening and closing of the cracks), both in the monotonic deformation process and in the cyclic convertible deformation process. The cracking or crushing mechanisms of the concrete result from the employed material softening rule which assumes a gradual loss of the material load-capacity till the attaining of the stressless state during the tension or compression processes.

Two cases of attaining the stressless states are distinguished: during the tension if $\sigma_0 < 0$ and during the compression if $\sigma_0 > 0$.

Attaining of the stressless state in the tension does not cause an irreversible failure of the material but only the cracking state. This state can be attained in the monotonic tension process or in the convertible unloading from the compression process that reverses the loading in the tension process. The cracking state does not reduce the compression strength and the redeformation process in the compression is possible after closing the generalised volumetric crack. The permanent volumetric deformation ε_0^* corresponding to the time instant of attaining the stressless state in the tension process can be recognised as the so-called volumetric limit strain. Thus, the crack opening state can be determined by the condition $\varepsilon_0 < \varepsilon_0^*$, whereas the crack closing state occurs when $\varepsilon_0 > \varepsilon_0^*$ making the recompressive deformation possible. The previously cracked material lost its tensile stress carrying capacity and a repeated crack opening occurs in the time instant of attaining the volumetric limit strain value ε_0^*.

Attaining of the stressless state in the compression process denotes, in accordance with the mean normal stress value, the crushing state of the concrete and the total loss of the stress carrying capacity if $\sigma_0 > \sigma_0^*$ or a semi-failure state of the concrete for low values of the mean normal compressive stress $0 < \sigma_0 < \sigma_0^*$, where σ_0^* is the conventional value of the mean normal stress which can be assumed as $\sigma_0^* = 0.25R_c$. The semi-failure state, being analogous to the cracking state, is characterised by the capability of a repeat beginning of the compression process if the current volumetric deformation is greater than the limit volumetric deformation $\varepsilon_0 > \varepsilon_0^*$; it is attained in the time instant of the semi-failure state of the concrete in the previous cycle of the deformation.

The assumed interpretation of the cracking and crushing mechanisms can be described by means of the indicator defined as the unitary active cross-section of the concrete,

$$\delta = \delta_c[\delta_0 + (1-\delta_0)\delta_t], \qquad \delta_0 = \begin{cases} 1 & \text{if } \sigma_0 \geq 0, \\ 0 & \text{if } \sigma_0 < 0, \end{cases} \tag{13}$$

where: $\delta_c = 1$ and $\delta_t = 1$ are the initial values of the partial indicators that change their values only once when the following conditions are fulfilled:

$$\begin{aligned} \delta_c &= 0 \quad \text{if } \sigma_0 \geq \sigma_0^* \text{ and } K = K_c < K_{\min}, \\ \delta_t &= 0 \quad \text{if } \sigma_0 < \sigma_0^* \text{ and } K = K_t < K_{\min}. \end{aligned} \tag{14}$$

Assuming that indicator δ can determine parameter K of the dynamic strength – plastic flow – material softening and the stress state in the concrete as follows: $\hat{K} = \delta K$ and $\hat{\sigma}_{ij} = \delta\sigma_{ij}$, the stressless state $\hat{\sigma}_{ij} = 0$, is determined directly by the indicator value $\delta = 0$ attained in the crushing process of the concrete $\delta_c = 0$ or in the cracking process $\delta_t = 0$ if $\sigma_0 < 0$.

2.2.2. Modified version of the model

In order to analyse the sensitivity of the numerical solution on the manner of the structure discretisation, a modification of the deformation model of the concrete was carried out. The modification is related to the only quantity that describes the model, namely to the limited strain ε_{uc} for the material softening phase of the deformation.

Now, it is assumed that the values of the limited strain ε_{uc} is the variable ε_{uce} and depends on the so-called scale parameters of the finite elements of the discretised model of the structure. As the parameters were taken: 1° the fracture energy G_f; 2° the equivalent length h_e of the finite element.

The value of the fracture energy G_f is determined on the basis of the experimental results, e.g. [1, 7]. In general, the fracture energy depends, among other things, on the type of the experiment (compression G_c, tension G_t, shearing G_s), on the class of the concrete and on the parameters that describe its strength and deformability as well as on the dimensions of the specimens under test.

The equivalent length h_e of the finite element generally depends on the type, dimensions and shape function of element as well as on the integration scheme. For the rectangular finite element of the area A_e, the equivalent length is determined in the following way,

$$h_e = \sqrt{A_e}\,. \tag{15}$$

Remaining the main idea of the formulation of the deformation model of the concrete and considering the stress–strain idealisation for uniaxial compression in this model (Fig. 1) as well as the introduced scale parameters of the finite elements, the limited strain ε_{uce} for the material softening phase of the deformation for each finite element can be determined as follows,

$$\varepsilon_{uce} = \frac{1}{1-K_{\min}^2}\left[\frac{2G_f}{h_e R_c} + \varepsilon_c^e - \left(1+K_{\min}^2\right)\varepsilon_{Rc}\right], \qquad \varepsilon_c^e = \frac{R_c}{E_c}, \qquad K_{\min} = \frac{R_t}{R_c}, \tag{16}$$

on the assumption that the fracture energy value $G_f \equiv G_c$ is a function of the total area under the stress–strain diagram.

On the other hand, in the case of the lack of experimental data, the fracture energy value can be determined on the basis of the approximation of the experimental stress–strain relation according to the methodology applied to the estimation of the limit strain values (16); using limit strains values ε_{Rc} and ε_{uc} (7) for the material softening phase of deformation, the fracture energy is then

$$G_f = \frac{h_p R_c}{2}\left[\left(1-K_{\min}^2\right)\varepsilon_{uc} + \left(1+K_{\min}^2\right)\varepsilon_{Rc} - \varepsilon_c^e\right], \tag{17}$$

where: h_p is the equivalent dimension of the concrete specimen subjected to the experimental investigations. Because of the lack of complete experimental results, the value $h_p = \sqrt[3]{V} = 15\,\mathrm{cm}$ was used in this paper as the dimension of the cubic specimen recommended by the Polish Standard for the investigation of the concrete strength.

3. Method of analysis

3.1. Discrete model of the reinforced concrete deep beam

The discretisation of the structure making use of the finite element method is based on:

1. the division of the structure on:
   - the finite elements of the matrix material (concrete elements),
   - the finite elements of the reinforcement bars material (slender steel bars distributed discretely in the matrix material);
2. the application of the matrix material and the reinforcement bars material joint-action hypothesis.

The matrix material (concrete) was divided in to rectangular elements to which the description of four-node elements with two degrees of freedom in each node was applied. A refinement of the division was made in the regions in which a concentration of stress was expected.

The reinforcement steel was modelled as linear bar elements to which the description of two-node elements with two degrees of freedom in each node was employed.

The concrete and reinforcement steel joint-action principle is realised in the following way:

1. two types of nodes are introduced:
   - the main element nodes, typical of the employed structure discretisation, when it is distinguished between the nodes of the matrix materialelements and those of the material elements of the reinforcement bars,
   - the common nodes of the matrix materialelements and the reinforcement bar material elements as selected from among the main nodes;
2. the displacement equivalence condition introduced in main nodes exclusively;

3. the computational Gauss points on the matrix materialelement area are introduced. This enables a full strain and stress state analysis and, in addition, the differentiation of the effort degree of the material within the simple element, as well as the modelling of the reinforcement steel and concrete joint-action within the dilatated and cracked areas.

3.2. Static equilibrium equations

The solution of the reinforced concrete deep beam static problem is reduced to the solution of the equilibrium equation system in the following incremental form,

$$K\,\Delta r_{i,i+1} = R_{i+1} - F_i\,, \tag{18}$$

where: K – elastic stiffness matrix, $\Delta r_{i,i+1}$ – searched increment of the generalised displacement vector of the system.

The generalised external loading vector in the 'i+1' step has the form

$$R_{i+1} = R_i + \Delta R_{i,i+1}\,, \tag{19}$$

whereas the generalised nodal forces vector, which corresponds to the stress state in the 'i' step, is defined as follows,

$$F_i = F_{i-1} + \Delta F_{i-1,i}, \qquad \Delta F_{i-1,i} = \int_V B^T \Delta\sigma_{i-1,i}\,\mathrm{d}V\,. \tag{20}$$

The system of equations (18), in the typical formulation of the initial loading method, is iteratively solved by the application of the modified Newton–Raphson method in that a single triangularisation of stiffness matrix K, which is constant during the loading process, is used.

4. Results of the numerical solutions

4.1. The subject of the analysis

Two directions of the analysis were considered; they were aimed at the demonstration of the objective of the introduced modification of the concrete model to a numerical modelling of the behaviour of the concrete and reinforced concrete structures.

The subject of the first direction of the analysis was a concrete deep beam (without reinforcement), while the subject of the second one was the reinforced concrete deep beam for which experimental results were given by Leonhardt and Walther [9]. In order to concentrate the attention, the present analysis concerns exclusively the static character of the loading.

4.2. Analysis of the concrete deep beam

The analysis concerns the rectangular concrete deep beam of dimensions 15 × 15 cm and of unit thickness.

The following material data of the concrete were taken to the analysis: compressive strength $R_c = 30$ MPa, tensile strength $R_t = 3$ MPa, modulus of elasticity $E_c = 25$ GPa. The fracture energy value was determined according to (17).

The concrete deep beam is subjected to a kinematic constrained deformation in the form of a permanent increment of the upper edge displacement.

Two manners of discretisation of the deep beam were applied, Fig. 2:

- the regular manner, with division into 5 × 5, 10 × 10 and 20 × 20 elements,
- the irregular manner, with division into 11 × 11 elements and with refinement inside the outer regions (11 × 11z) or with refinement inside the inner regions (11 × 11w).

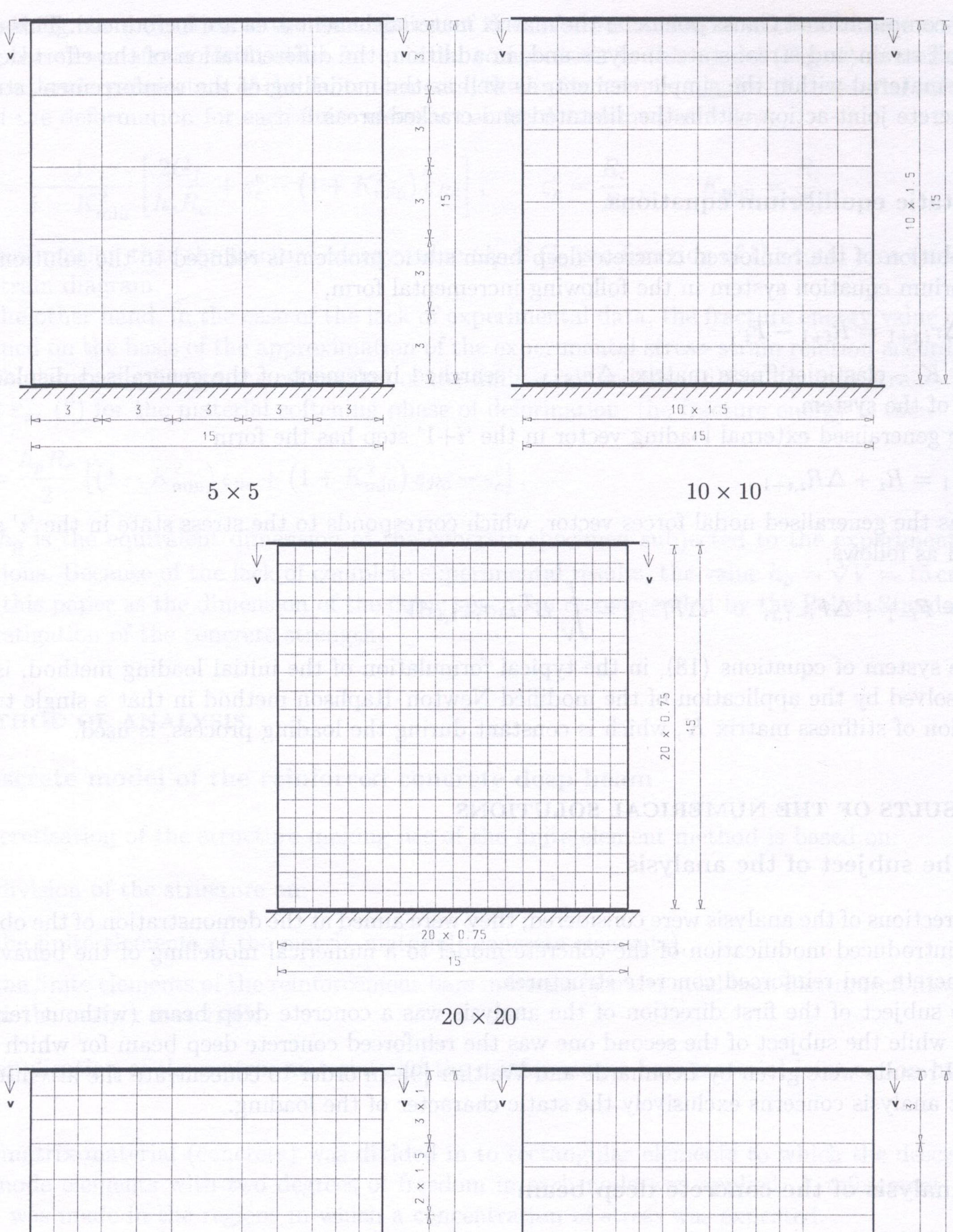

Fig. 2. Discretisation of concrete deep beam

Figure 3 shows the results of the numerical solution for the regular manner of discretisation of the deep beam using a basic version of the model of the concrete in which the scale parameters of the finite elements were disregarded. In the Figure, the relationships between total support reaction **R** versus displacement **v** of the upper edge of the deep beam are shown. The total support reaction is treated as a sum of the finite element nodal forces on the bottom edge of the deep beam. The results indicate that the number of the element division has no influence on the results of the numerical solution for the elastic range of deformation. However, depending on the manner of the finite element division of deep beam, there is obtained an unimportant differentiation of the maximal reaction value (the so-called load carrying capacity of the deep beam) and, first of all, a particularly essential differentiation of the results for the range of the post-critical behaviour in the phase of the material softening is observed.

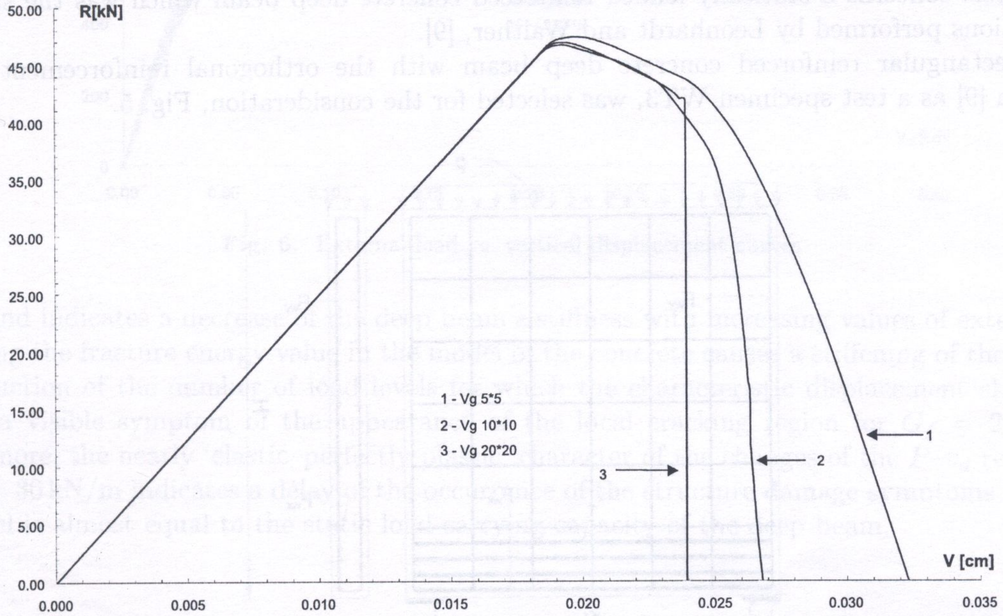

Fig. 3. Vertical displacement vs. support reaction curves

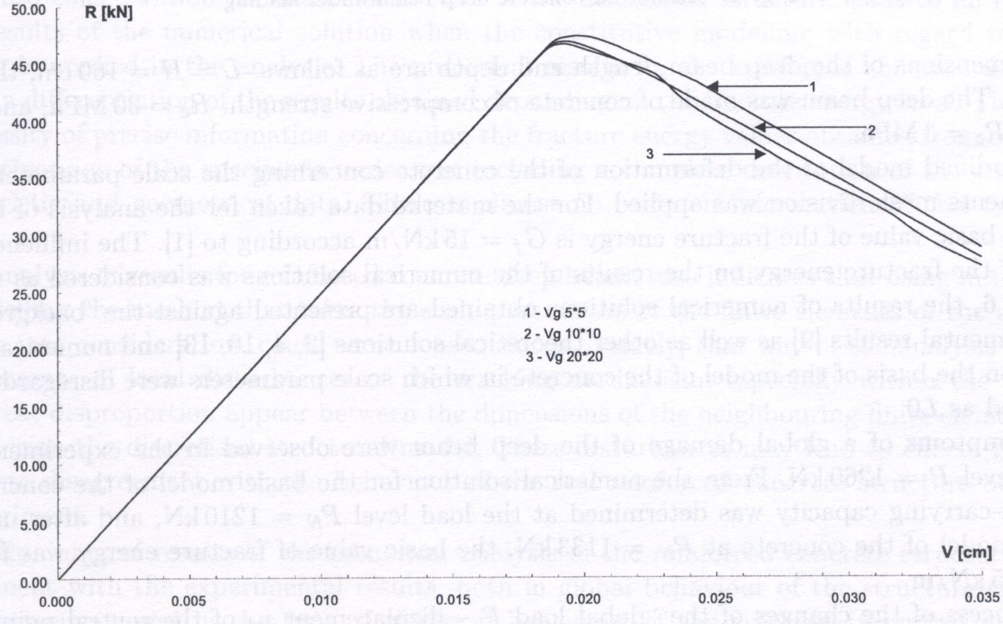

Fig. 4. Vertical displacement vs. support reaction curves

In turn, in Fig. 4 analogous relationships, obtained by using the modified version of the model of the concrete with scale parameters, are shown. In this case, the quality changes of the behaviour of the discretised model of the deep beam in the post-critical range are observed. Thus, a good agreement of the results in this range occurs, and a better convergence of the results may be achieved by mesh refinement of the deep beam.

In the case of the irregular manner of the division of the deep beam, results of the numerical solutions demonstrating a similar character of the behaviour of the discretised model of the structure were obtained.

4.3. Analysis of the reinforced concrete deep beam

The analysis concerns a statically loaded reinforced concrete deep beam which was the subject of investigations performed by Leonhardt and Walther, [9].

The rectangular reinforced concrete deep beam with the orthogonal reinforcement system, marked in [9] as a test specimen WT3, was selected for the consideration, Fig. 5.

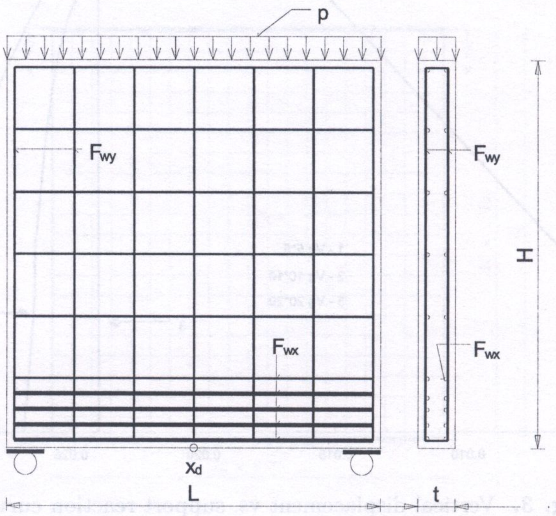

Fig. 5. Reinforced concrete deep beam under loading

The dimensions of the deep beam: length and depth are as follows: $L = H = 160$ cm, thickness: $t = 10$ cm. The deep beam was made of concrete of compressive strength: $R_c = 30$ MPa, and tensile strength: $R_t = 3$ MPa.

The modified model of the deformation of the concrete concerning the scale parameters of the finite elements mesh division was applied. For the material data taken for the analysis of the deep beam, the basic value of the fracture energy is $G_f = 15$ kN/m according to [1]. The influence of the changes of the fracture energy on the results of the numerical solutions was considered as well.

In Fig. 6, the results of numerical solutions obtained are presented against the 'background' of the experimental results [9] as well as other theoretical solutions [2, 4, 10, 13] and numerical results obtained on the basis of the model of the concrete in which scale parameters were disregarded; they are denoted as $L0$.

The symptoms of a global damage of the deep beam were observed in the experiment [9] at the load level $P = 1260$ kN. From the numerical solution for the basic model of the concrete, the static load-carrying capacity was determined at the load level $P_N = 1210$ kN, and after using the modified model of the concrete at $P_N = 1133$ kN, the basic value of fracture energy was found to be $G_f = 15$ kN/m.

The process of the changes of the 'global load P – displacement v_d of the central point at the bottom edge of deep beam' relationship is different in comparison to the basic results (denoted

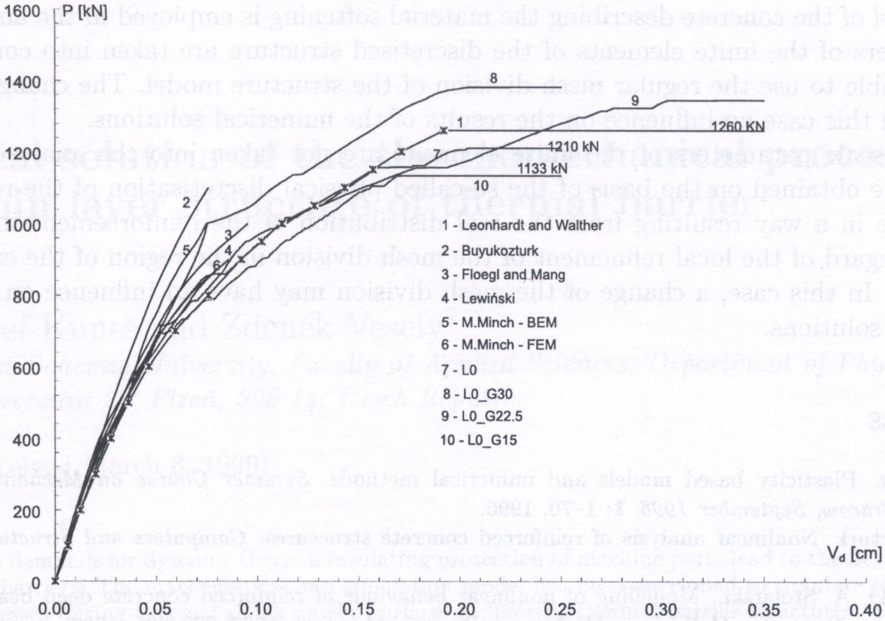

Fig. 6. External load vs. vertical displacement curves

as $L0$) and indicates a decrease of the deep beam's stiffness with increasing values of external load. Increasing the fracture energy value in the model of the concrete causes a stiffening of the structure and reduction of the number of load levels for which the characteristic displacement skips occur. This is a visible symptom of the appearance of the local cracking region for $G_f = 22.5\,\mathrm{kN/m}$. Furthermore, the nearly 'elastic–perfectly plastic' character of the changes of the P–v_d relationship for $G_f = 30\,\mathrm{kN/m}$ indicates a delay of the occurrence of the structure damage symptoms before the load level is almost equal to the static load-carrying capacity of the deep beam.

5. Conclusions

Taking into consideration the scale parameters of the discretised structure leads to an objectivity of the results of the numerical solution when the constitutive modelling with regard to material softening is applied in the analysis. The numerical results for the reinforced concrete deep beams indicate a differentiation of the results obtained depending on the fracture energy level. That implies the necessity of precise information concerning the fracture energy values obtained from experimental investigations of the specimens under test including a complete description of the stress–strain characteristic and geometrical data. There is almost no common information of this type at the moment.

The analysis of results for reinforced concrete deep beams also indicates that using in the numerical modelling the method that takes the scale parameters of the finite elements of the discretised structure into consideration, cause some consequences. Namely, that way of the analysis influences the occurrence of local disturbances of the cracking mechanism, especially within the regions in that a great disproportion appear between the dimensions of the neighbouring finite elements of the mesh division the discretised structural model. These disturbances may lead to effects that are in a complete disagreement with the behaviour of the real reinforced concrete structure observed in the experiment.

The obtaining of results of the numerical analysis of the reinforced concrete structure that are in agreement with the experimental results, both in global behaviour of the structure type of the load–displacement and in the type of the local behaviour of the cracking (fracture) mechanism, requires a proper choice the discretisation manner.

If the model of the concrete describing the material softening is employed in the analysis and the scale parameters of the finite elements of the discretised structure are taken into consideration, it is more advisable to use the regular mesh division of the structure model. The change of the mesh division has in this case no influence on the results of the numerical solutions.

But if the scale parameters of the finite elements are not taken into the analysis, an efficient solution can be obtained on the basis of the so-called physical discretisation of the reinforced concrete structure in a way resulting from the real distribution of the reinforcement in the concrete matrix with regard of the local refinement of the mesh division in the region of the expected stress concentration. In this case, a change of the mesh division may have an influence on the results of the numerical solutions.

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