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# A backup orientation system based on inverse problems technique

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The correct orientation of the spacecraft during a space flight is one of the main tasks of astronautics, because its orientation affects the correct operation of all subsystems, in particular power supply systems, where the orientation of solar panels is one of the key tasks. From this point of view, the tasks of developing backup or replacement of orientation systems are becoming relevant. In this paper, an alternative for the development of an orientation sensor is proposed, which is sensitive to radiation heating from the Sun or other celestial bodies that emit strongly. Thus, the tasks of developing a heat flux sensor responsive to changes in heat fluxes become relevant. This article is devoted to the analysis of technical capabilities to create such a sensor.

**Keywords:** computational methods, measurement and instrumentation, radiation, inverse problems, orbital spacecraft, orientation.

## 1. Introduction

One of the main problems encountered in spacecraft design is the development of control systems and particularly the orientation system that provides angular directions to the space objects (Sun, planets, stars). A promising way to develop such systems is based on measuring the radiative flux from the environment. Such an approach has been suggested for the first time in [4]. Unfortunately, in the majority of practical situations, the direct measurements of heat flux are problematic. These difficulties can be overcome with the use of some indirect thermal measurements combined with an inverse problem technique. The problem of estimating the angular orientation of a spacecraft requires solving two inverse problems sequentially. The first one is estimating heat fluxes absorbed by the spacecraft surface. The second one is determining angles of orientation based on the estimated values of radiative heat fluxes.

In the general case, the orientation of a surface element of spacecraft is determined by the following nine angles:

1) Three angles determine the relative position of the equatorial $XYZ$ and orbital coordinate systems: $\Omega$ is the longitude of ascending node, $i$ is the inclination of orbit, and $u$ is the argument of latitude (Fig. 1a). The planetocentric equatorial coordinate system can be considered as an inertial coordinate system for the most engineering problems. These angles are known from the spaceflight program.

2) Direction angles $\alpha_N$, $\beta_N$, $\gamma_N$ for the vector $\overline{N}$ of the orientation of the plane in the coordinate system $X_C Y_C Z_C$ associated with the spacecraft (Fig. 1b). These angles are known from the spacecraft design.

3) Three angles determine the relative position of the orbital coordinate systems as well as the spacecraft coordinate systems. These are: the pitch angle $\vartheta$, the yaw angle $\psi$, and the angle of heel $\gamma$. These angles provide the orientation of the spacecraft during the flight.

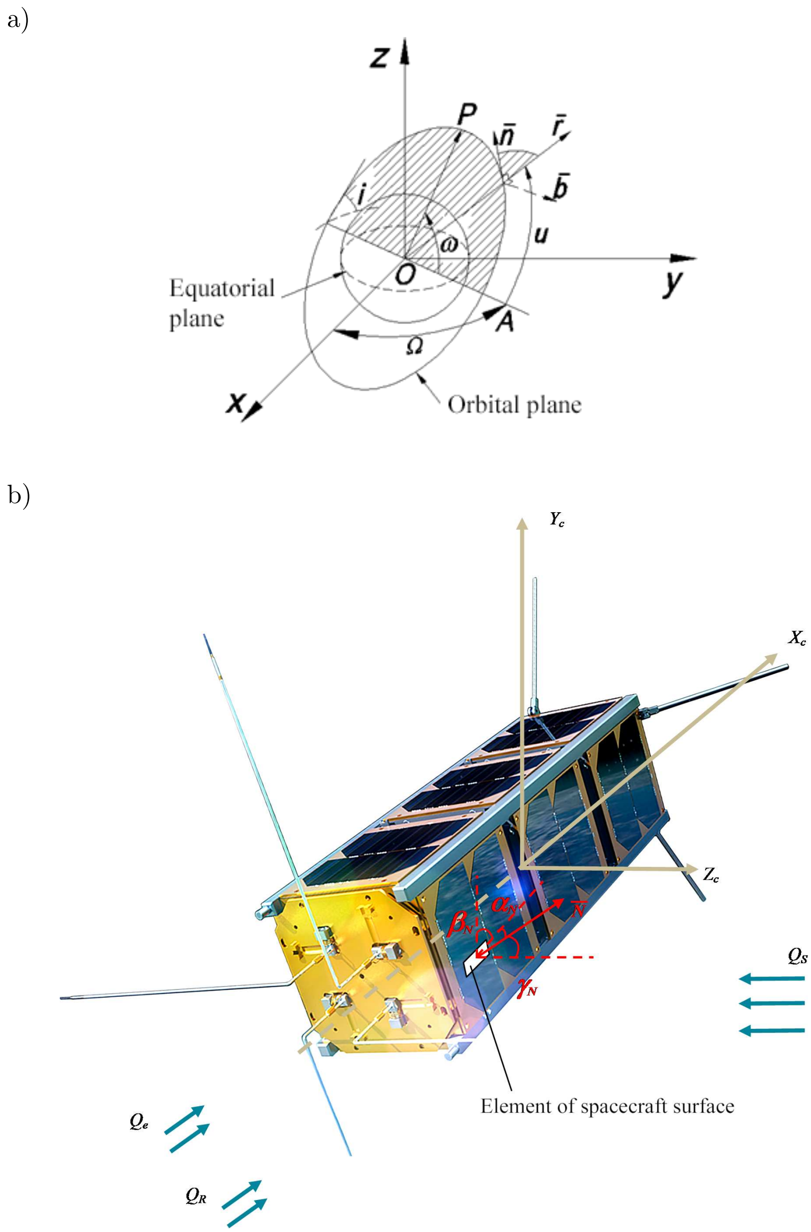


**Fig. 1.** The orientation of the surface element: a) in the equatorial and orbital coordinate systems, b) in the coordinate system associated with the spacecraft.

One can use the transition matrix from the coordinate system associated with the spacecraft to the orbital coordinate system (Fig. 1b) to determine the vector $\overline{N}$.

Therefore, one can define the orientation of the analysed element in space by using nine specified angles. We also need to know the spacecraft orbit parameters to determine the position of the spacecraft: the apocenter altitude, the pericenter altitude, and the ascending node-pericenter angle (for the elliptical orbit).

Heat transfer in heat flux sensors installed on the spacecraft surface can be analysed by two different approaches:

1) The lumped parameters system is

$$d_m \rho_m c_m \frac{dT_m}{d\tau} = A_{sm}(q_{sm}(\tau) + q_{Rm}(\tau)) + \varepsilon_m q_{em}(\tau) - \varepsilon_m \sigma T_m^4, \qquad m = 1, 2, .., M, \tag{1}$$

$$T_{ml}(\tau_{\min}) = T_{l0}, \qquad m = 1, 2, ..., M, \tag{2}$$

where $m$ is number of sensor, $M$ is total number of sensors, $d_m$, $\rho_m$, $c_m$, $A_{sm}$, $\varepsilon_m$ are the thickness, the density, the heat capacity, the absorptivity and emissivity of an optically gray $m$-th sensor, respectively, $q_{sm}(\Omega, i, u, \alpha_{Nm}, \beta_{Nm}, \gamma_{Nm}, \psi, \vartheta, \gamma)$ is the integral (over the spectrum) solar radiative flux, $q_{Rm}(\Omega, i, u, \alpha_{Nm}, \beta_{Nm}, \gamma_{Nm}, \psi, \vartheta, \gamma)$ is the integral solar radiative flux reflected by a planet, and $q_{em}(\Omega, i, u, \alpha_{Nm}, \beta_{Nm}, \gamma_{Nm}, \psi, \vartheta, \gamma)$ is the integral radiative flux emitted by a planet.

2) The distributed parameters system:

$$\rho_m c_m (T_m(\tau, x)) \frac{\partial T_m(\tau, x)}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_m (T_m(\tau, x)) \frac{\partial T_m(\tau, x)}{\partial x}\right), \\ 0 < x < d_m, \qquad m, \tau_{\min} \le \tau \le \tau_{\max}, \qquad m = 1, 2, ..., M, \tag{3}$$

$$T_m(\tau_{\min}, x) = T_{l0}(x), \qquad 0 \le x \le d_m, \qquad m = 1, 2, ..., M, \tag{4}$$

$$-\lambda_m (T(0, \tau)) \frac{\partial T_m(0, \tau)}{\partial x} = A_{sm}(q_{sm} + q_{Rm}) + \varepsilon_m q_{em} - \varepsilon_m \sigma T_m^4(0, \tau), \tag{5}$$

$$-\lambda_m (T_m(d_m, \tau)) \frac{\partial T_m(X_m, \tau)}{\partial x} = q_{2m}(\tau), \tag{6}$$

where $\lambda_m$ is the thermal conductivity of $m$-th sensors.

In the first case, one should solve an ill-posed problem [5] of differentiation of experimental function $T$:

$$q_m^{\exp} = d_m \rho_m c_m \frac{dT_m}{d\tau + \varepsilon_m \delta T_m^4}, \qquad m = 1, 2, ..., M. \tag{7}$$

In the second case, the boundary inverse heat transfer problem should be solved [1]:

$$q_m^{\exp} = -\lambda_m (T(0, \tau)) \frac{\partial T_m(0, \tau)}{\partial x} + \varepsilon_m \sigma T_m^4(0, \tau). \tag{8}$$

Therefore, we can find some estimates for the heat flux absorbed by the heat flux sensor:

$$q_m^{\exp} = A_{sm}(q_{sm} + q_{Rm}) + \varepsilon_m q_{em} - \varepsilon_m \sigma T_m^4(0, \tau), \qquad m = 1, 2, ...M, \tag{9}$$

which can be used then to estimate angles $\psi$, $\vartheta$, $\gamma$ using adequate calculated models for

$$q_{sm}(\Omega, i, u, \alpha_{Nm}, \beta_{Nm}, \gamma_{Nm}, \psi, \vartheta, \gamma),$$

$$q_{Rm}(\Omega, i, u, \alpha_{Nm}, \beta_{Nm}, \gamma_{Nm}, \psi, \vartheta, \gamma),$$

$$q_{em}(\Omega, i, u, \alpha_{Nm}, \beta_{Nm}, \gamma_{Nm}, \psi, \vartheta, \gamma)$$

for $m = 1, 2, ..., M$.

## 2. Thermobalistic analysis of orbital space flight

### 2.1. Solar radiative flux

The calculation of the surface heating by direct solar radiation is relatively simple:

$$q_s = S \cdot \cos\beta, \tag{10}$$

where $\beta$ is the angle between the normal to the surface element $\overline{N}$ and the direction to the Sun $\overline{S}$, $S$ is the solar radiative flux incident normal to the unit surface at the outer edge of the atmosphere at an average distance of the planet from the Sun:

$$S = \frac{S_0}{L^2}, \tag{11}$$

where $L$ is the average distance of the planet from the Sun in AU and $S_0$ = 1398 W/m$^2$ is the solar constant for the Earth. In the case of $\gamma > 90°$, solar radiation cannot reach the surface under consideration and $q_s = 0$.

### 2.2. Solar radiation reflected from the planet

The solar radiation flux reflected from the planet and falling on the spacecraft surface element depends on the geophysical properties of the planet surface and the atmosphere ($A_{av}$), and the position of the element relative to the direction and physical model of solar radiation reflection ($\varphi_2$) [2, 3] is

$$q_R = A_{av} S \varphi_2, \tag{12}$$

where $A_{av}$ is the planet albedo, and $\varphi_2$ is the angular combined coefficient. The system of the planet – the Sun surface element, characterized by the parameters $\theta_0$, $\gamma_S$, $\delta_S$, $\psi_n$, is shown in Fig. 2,

$$\theta_0 = \arcsin\frac{\overline{R}}{\overline{R}+H}. \tag{13}$$

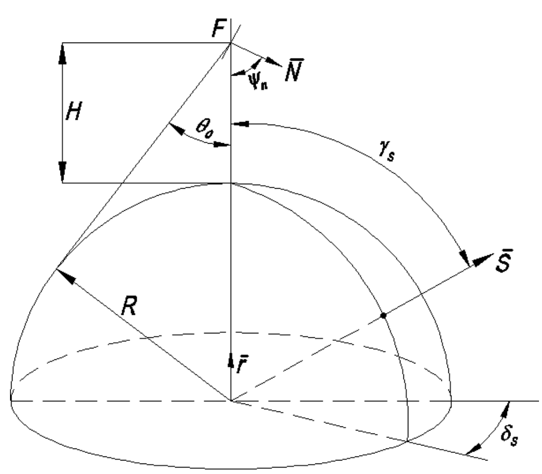


**Fig. 2.** The main parameters for angular coefficients calculation.

The diffuse reflection model is valid for values $\gamma_S \le 60°$ when the reflection of solar radiation by a planet satisfies Lambert's law $q_R|_{\gamma_S \le 60°} = q_R^D$. On the contrary, the mirror reflection model $q_R|_{\gamma_S > 60°} = q_R^M$ is valid for values $\gamma_S > 60°$.

In the case of diffuse reflection, the solar radiative flux reflected by the planet is determined as follows:

$$q_R^D = A_{av} S \varphi_2, \tag{14}$$

$$\varphi_2 = f_2^*(\theta_0, \psi_n)\cos\gamma_S + f_3^*(\theta_0, \psi_n)\sin\psi_n \sin\gamma_S \cos\delta_S, \tag{15}$$

where

$$f_2^*(\theta_0, \psi_n) \approx \frac{f_2(\theta_0)}{\sin^2\theta_0}\varphi_1(\theta_0, \psi_n),$$

$$f_2(\theta_0) = \frac{1}{4}\left(1 + \sin^2\theta_0 + 2\sin^3\theta_0 + \frac{\cos^4\theta_0}{2\sin\theta_0}\ln\frac{1-\sin\theta_0}{1+\sin\theta_0}\right),$$

$$f^*(\theta_0, \psi_n) = \begin{cases} f_3(\theta_0), & \text{if } 0 \le \psi_n \le \frac{\pi}{2} - \theta_0, \\ f_3(\theta_0)\frac{\theta_0 + \frac{\pi}{2} - \psi_n}{2\theta_0}, & \text{if } \frac{\pi}{2} - \theta_0 \le \psi_n \le \frac{\pi}{2} + \theta_0, \end{cases}$$

$$f_3(\theta) = \frac{\cos^2\theta_0\left(3 + \sin^2\theta_0\right)}{16\sin\theta_0}\ln\frac{1+\sin\theta_0}{1-\sin\theta_0} - \frac{(1-\sin\theta_0)\left(3 + 3\sin\theta_0 + 2\sin^2\theta_0\right)}{8}.$$

Note that the approximate equation for the $\varphi_2$ formula can give small negative values of radiative flux at large values of $\gamma_S$ and $\delta_S > \frac{\pi}{2}$. In these cases, the radiative flux should be taken equal to zero.

The reflected radiative flux at $\gamma_S > 60°$ is determined as follows:

$$q_R^M = A_{av} \cdot S \cdot F^R \cdot k, \tag{16}$$

where $F^R = \cos(\psi_n - (2\beta - \gamma_S))$ is the unit area of the midsection of spacecraft surface, calculated from the propagation direction of the mirror reflected radiation, and

$$k = \frac{b_0^2 \sin 2\beta}{2\sin\gamma_S\left[2\cos(2\beta - \gamma_S) - b_0\cos\beta\right]}$$

is the scattering coefficient of the homogeneous flux of radiative energy in mirror reflection from the spherical surface:

$$b_0 = \frac{\overline{R}}{\overline{R} + H}. \tag{17}$$

$\beta$ is the angle of reflection of incident radiation from the planet surface. The relation between the angles $\beta$ and $\gamma_S$ is:

$$b_0 \sin\beta = \sin(2\beta - \gamma_S). \tag{18}$$

In general, the spacecraft moving along a stationary orbit appears periodically in the shadow of the planet. When the spacecraft is in the shadow ($\gamma_S > (\pi - \theta_0)$), the values of $q_S$ and $q_R$ become equal to zero.

### 2.3. Self-emission of the planet

The planet is considered as a diffusely emitting spherical body with an effective radius of $\overline{R} = R_0 + H_a$, where $R_0$ is the average radius of the planet and $H_a$ is the upper boundary of the effective radiated atmospheric layer. All the planets are divided into three types depending on the character of distribution of the planet's surface own radiation emission. For planets of the first type (Earth, Venus, Jupiter) their own radiation is assumed constant over the surface:

$$q_e = \frac{1 - A_{av}}{4} S\varphi_1, \tag{19}$$

where $\varphi_1$ is the angular coefficient between the element of spacecraft and the planet, which can be calculated as:

$$\varphi_1 = \begin{cases} \cos\psi_n \sin^2\theta_0, \qquad \text{if} \quad 0 \le \psi_n \le \dfrac{\pi}{2} - \theta_0, \\ \dfrac{\cos\psi_n \sin^2\theta_0}{\pi}\left[\dfrac{\pi}{2} + \arcsin(\text{ctg}\theta_0 \text{ctg}\psi_n)\right] + \dfrac{1}{\pi}\arcsin\dfrac{\sqrt{\sin^2\theta_0 - \cos^2\psi_n}}{\sin\psi_n} \\ \qquad -\dfrac{1}{\pi}\cos\theta_0\sqrt{\sin^2\theta_0 - \cos^2\psi_n}, \qquad \text{if} \quad \dfrac{\pi}{2} - \theta_0 \le \psi_n \le \dfrac{\pi}{2} + \theta_0, \\ 0, \qquad \text{if} \quad \dfrac{\pi}{2} + \theta_0 \le \psi_n \le \pi. \end{cases} \tag{20}$$

## 3. Geometry inverse problem

To define the orientation of the spacecraft, we need three angles: the pitch angle ($\vartheta$), the yaw angle ($\psi$), and the angle of heel ($\gamma$). If a few heat flux sensors are installed on the spacecraft surface, the geometric inverse heat transfer problem can be formulated as follows: to determine three unknown angles $\vartheta$, $\psi$, and $\gamma$ from a set of nine angles, which characterize the orientation of spacecraft, by indirect measurements of radiative heat fluxes absorbed by the sensors. In the case of small satellite (standard CubeSat) of simple shape, six sensors can be installed at each surface of the spacecraft (Fig. 1a). In the case of the same radiative properties of all the sensors, the mathematical statement for the inverse problem can be formulated as follows:

$$\begin{cases} q_{\Sigma 1}(\vartheta,\psi,\gamma) = A_{sm}q_{S1}(\vartheta,\psi,\gamma) + A_{sm}q_{R1}(\vartheta,\psi,\gamma) + \varepsilon_m q_{e1}(\vartheta,\psi,\gamma); \\ q_{\Sigma 2}(\vartheta,\psi,\gamma) = A_{sm}q_{S2}(\vartheta,\psi,\gamma) + A_{sm}q_{R2}(\vartheta,\psi,\gamma) + \varepsilon_m q_{e2}(\vartheta,\psi,\gamma); \\ \ldots \\ q_{\Sigma m}(\vartheta,\psi,\gamma) = A_{sm}q_{Sm}(\vartheta,\psi,\gamma) + A_{sm}q_{Rm}(\vartheta,\psi,\gamma) + \varepsilon_m q_{em}(\vartheta,\psi,\gamma), \end{cases} \tag{21}$$

where $q_{\Sigma m}$ is the integral heat flux absorbed by $m$-th sensor.

Then the inverse problem can be formulated as solving the following system of equations:

$$q_m^{\text{exp}} \cong q_{\Sigma m}(\vartheta,\psi,\gamma), \qquad m = 1, 2, \ldots, M. \tag{22}$$

One cannot obtain the analytical solution for (22) because of its essential non-linearity and complex transcendence. Therefore, this problem should be solved numerically. The searched vector can be found using the least square method. The corresponding residual functional is as follows:

$$J_{\min} = \sum_{m=1}^{M} \left(q_{\Sigma m}(\vartheta,\psi,\gamma) - q_m^{\text{exp}}\right)^2. \tag{23}$$

The sufficient condition of a minimum of functional (23) is its gradient equal to zero. As a result, (23) is reduced to the following coupled equations:

$$\begin{cases} \dfrac{\partial J}{\partial \vartheta} = 0, \\ \dfrac{\partial J}{\partial \psi} = 0, \\ \dfrac{\partial J}{\partial \gamma} = 0. \end{cases} \tag{24}$$

Using (23), we can rewrite (24) as follows:

$$\begin{cases} 2\sum\limits_{m=1}^{M} \left(q_{\Sigma m}(\vartheta,\psi,\gamma) - q_m^{\exp}\right) \dfrac{\partial q_{\Sigma m}}{\partial \vartheta} = 0, \\ 2\sum\limits_{m=1}^{M} \left(q_{\Sigma m}(\vartheta,\psi,\gamma) - q_m^{\exp}\right) \dfrac{\partial q_{\Sigma m}}{\partial \psi} = 0, \\ 2\sum\limits_{m=1}^{M} \left(q_{\Sigma m}(\vartheta,\psi,\gamma) - q_m^{\exp}\right) \dfrac{\partial q_{\Sigma m}}{\partial \gamma} = 0, \end{cases} \tag{25}$$

where the partial derivatives can be calculated as:

$$\frac{\partial q_{\Sigma m}}{\partial \vartheta} = A_{sm}\frac{\partial q_{Sm}}{\partial \vartheta} + A_{sm}\frac{\partial q_{Rm}}{\partial \vartheta} + \varepsilon_m \frac{\partial q_{em}}{\partial \vartheta}, \tag{26}$$

$$\frac{\partial q_{\Sigma m}}{\partial \psi} = A_{sm}\frac{\partial q_{Sm}}{\partial \psi} + A_{sm}\frac{\partial q_{Rm}}{\partial \psi} + \varepsilon_m \frac{\partial q_{em}}{\partial \psi}, \tag{27}$$

$$\frac{\partial q_{\Sigma m}}{\partial \gamma} = A_{sm}\frac{\partial q_{Sm}}{\partial \gamma} + A_{sm}\frac{\partial q_{Rm}}{\partial \gamma} + \varepsilon_m \frac{\partial q_{em}}{\partial \gamma}. \tag{28}$$

The conjugate gradient method provides the most effective solving of this problem:

$$\xi^{(k+1)} = \xi^{(k)} - \alpha_k p^{(k)}, \tag{29}$$

where $\xi = (\vartheta, \psi, \gamma)$, $k$ is iteration number, and $p^{(k)}$ is descent direction for $k$-th iteration:

$$p^{(k)} = \operatorname{grad} J(\xi^{(k)}) + \beta_k p^{(k-1)}, \tag{30}$$

$$\beta_k = \frac{\left|\operatorname{grad} J\left(\xi^{(k)}\right)\right|^2}{\left|\operatorname{grad} J\left(\xi^{(k-1)}\right)\right|^2} = \frac{\sum\limits_{i=1}^{3}\left[\dfrac{\partial J\left(\xi^{(k)}\right)}{\partial \xi_i}\right]^2}{\sum\limits_{i=1}^{3}\left[\dfrac{\partial J\left(\xi^{(k-1)}\right)}{\partial \xi_i}\right]^2}. \tag{31}$$

The initial value of descent direction for $k = 0$:

$$p^{(0)} = \operatorname{grad} J(\xi^{(0)}) \quad \text{for} \quad \beta_0 = 0, \tag{32}$$

where $\alpha_k$ – step of descent at the $k$ iteration, which is determined by the following procedure. We give the increment of the function $\Delta q_{\Sigma m}(\vartheta,\psi,\gamma)$ in equation (23), i.e., we have

$$J = \sum_{m=1}^{M} \left(q_{\Sigma m}(\vartheta,\psi,\gamma) + \Delta q_{\Sigma m}(\vartheta,\psi,\gamma) - q_m^{\exp}\right)^2, \tag{33}$$

where

$$\Delta q_{\Sigma m}(\vartheta,\psi,\gamma) = \alpha_{k1}\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right) + \alpha_{k2}\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right) + \alpha_{k3}\frac{\partial q_{\Sigma m}}{\partial \gamma}\left(-p_{\gamma}^{(k)}\right), \tag{34}$$

where $p_{\vartheta}^{(k)}$, $p_{\psi}^{(k)}$, $p_{\gamma}^{(k)}$ are components of a vector $p^{(k)}$.

Opening brackets (33) as the square of the sum, we obtain:

$$J = \sum_{m=1}^{M}\left(q_{\Sigma m}(\vartheta,\psi,\gamma) + \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)\right)^2 - 2\sum_{m=1}^{M}\left(q_{\Sigma m}(\vartheta,\psi,\gamma) + \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)\right)q_m^{\exp} + \sum_{m=1}^{M}(q_m^{\exp})^2. \tag{35}$$

Taking the derivative of equation (35) with respect to $\alpha_k$ and equate the resulting expression to zero to determine the roots of the equation:

$$\frac{\partial J}{\partial \alpha_k} = 2\sum_{m=1}^{M}\left(q_{\Sigma m}(\vartheta,\psi,\gamma) + \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)\right) \times \frac{\partial \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)}{\partial \alpha_k} - 2\sum_{m=1}^{M}\frac{\partial \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)}{\partial \alpha_k}q_m^{\exp} = 0, \tag{36}$$

where

$$\frac{\partial \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)}{\partial \alpha_k}$$

is the derivative of the increment of the function $\Delta q_{\Sigma m}(\vartheta,\psi,\gamma)$ with respect to the components $\alpha_k$:

$$\frac{\partial \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)}{\partial \alpha_{k1}} = \frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right), \tag{37}$$

$$\frac{\partial \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)}{\partial \alpha_{k2}} = \frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right), \tag{38}$$

$$\frac{\partial \Delta q_{\Sigma m}(\vartheta,\psi,\gamma)}{\partial \alpha_{k3}} = \frac{\partial q_{\Sigma m}}{\partial \gamma}\left(-p_{\gamma}^{(k)}\right). \tag{39}$$

Simplifying equation (36) and taking (34), (37)–(39), we obtain a system of equations:

$$\alpha_{k1}\left[\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right)\right]^2 + \alpha_{k2}\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right)\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right) + \alpha_{k3}\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial \gamma}\left(-p_{\gamma}^{(k)}\right)\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right) = \sum_{m=1}^{M}q_m^{\exp}\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right) - \sum_{m=1}^{M}q_{\Sigma m}(\vartheta,\psi,\gamma)\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right), \tag{40}$$

$$\alpha_{k1}\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial \vartheta}\left(-p_{\vartheta}^{(k)}\right)\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right) + \alpha_{k2}\sum_{m=1}^{M}\left[\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right)\right]^2 + \alpha_{k3}\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial \gamma}\left(-p_{\gamma}^{(k)}\right)\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right) = \sum_{m=1}^{M}q_m^{\exp}\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right) - \sum_{m=1}^{M}q_{\Sigma m}(\vartheta,\psi,\gamma)\frac{\partial q_{\Sigma m}}{\partial \psi}\left(-p_{\psi}^{(k)}\right), \tag{41}$$

$$\alpha_{k1}\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial\vartheta}\left(-p_{\vartheta}^{(k)}\right)\frac{\partial q_{\Sigma m}}{\partial\gamma}\left(-p_{\gamma}^{(k)}\right)+\alpha_{k2}\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial\psi}\left(-p_{\psi}^{(k)}\right)\frac{\partial q_{\Sigma m}}{\partial\gamma}\left(-p_{\gamma}^{(k)}\right)$$
$$+\alpha_{k3}\left[\sum_{m=1}^{M}\frac{\partial q_{\Sigma m}}{\partial\gamma}\left(-p_{\gamma}^{(k)}\right)\right]$$
$$=\sum_{m=1}^{M}q_m^{\exp}\frac{\partial q_{\Sigma m}}{\partial\gamma}\left(-p_{\gamma}^{(k)}\right)-\sum_{m=1}^{M}q_{\Sigma m}(\vartheta,\psi,\gamma)\frac{\partial q_{\Sigma m}}{\partial\gamma}\left(-p_{\gamma}^{(k)}\right). \quad (42)$$

The system of equations (40)–(42) is linear. This system can be written in the form:

$$A\overline{x}=\overline{b}, \quad (43)$$

where $A$ is the symmetric quadratic matrix, i.e., $A=(a_{ij})=A^T$, and $a_{ij}$ are components of the unknown terms $\alpha_{k1}$, $\alpha_{k2}$ and $\alpha_{k3}$, $\overline{b}=(b_1,b_2,b_3)^{\mathrm{T}}$ is the vector of the right parts of the system, and $\overline{x}=(\alpha_{k1},\alpha_{k2},\alpha_{k3})^{\mathrm{T}}$ is the vector-column of unknown values.

The method of solving the system (40)–(42) is the square root method.

The optimization process terminates when

$$\left|\operatorname{grad}J\left(\xi^{(k)}\right)\right|=\left\{\sum_{i=1}^{3}\left[\frac{\partial J\left(\xi^{(k)}\right)}{\partial\xi_i}\right]^2\right\}^{1/2}\leq\varepsilon, \quad (44)$$

where $\varepsilon$ is the measurement error.

In order to run the optimization algorithm using the conjugate gradient method, it is necessary to specify the initial approximation of the unknown angles $\vartheta_0$, $\psi_0$ and $\gamma_0$, which are chosen arbitrarily.

The search for the global extremum using the conjugate gradient method by setting the initial approximations does not lead to the required results because the residual functional has several extremums (Figs 3–5). Figures 3–5 show that the extremums are well separated from each other, and the method of local optimization converges to local extremums (Figs 6–9). Therefore, to determine a global extremum we will use the method of random restarts.

Firstly, three random numbers distributed uniformly on the interval [0, 360] are generated. These three generated numbers are specified respectively as the initial approximation of the unknown angles $\vartheta_0$, $\psi_0$ and $\gamma_0$. Secondly, the conjugate gradient method is run to determine the local extremum and the value of the residual functional by using the obtained initial approximation.

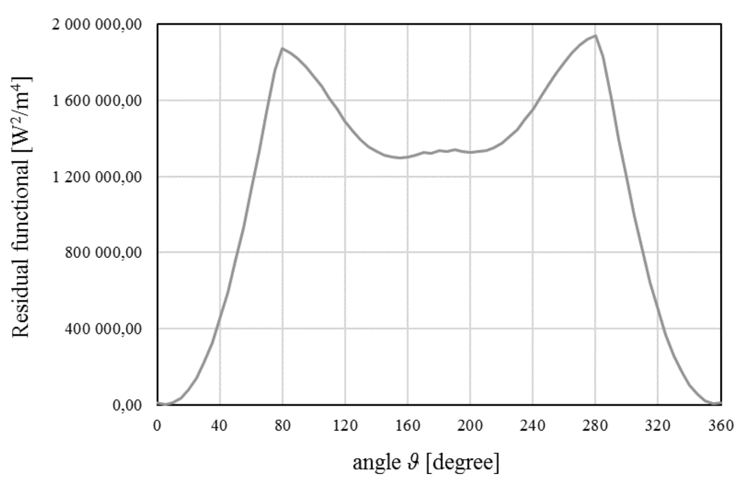


**Fig. 3.** Cross-section of the residual functional by the angle $\vartheta$ and $\psi=70°$, $\gamma=20°$.

Repeating the previous steps by the definitions of the local extremum and the value of the residual functional, we obtain the following parameter vector:

$$\begin{pmatrix} \vartheta_1 & \psi_1 & \gamma_1 & J_1 \\ \vartheta_2 & \psi_2 & \gamma_2 & J_2 \\ \ldots & & & \\ \vartheta_j & \psi_j & \gamma_j & J_j \end{pmatrix}, \tag{45}$$

where $j$ is the number of restarts.

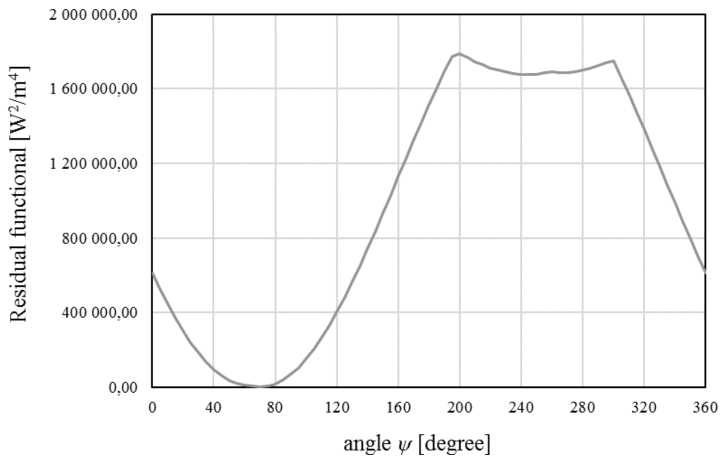


**Fig. 4.** Cross-section of the residual functional by the angle $\psi$ and $\vartheta = 5°$, $\gamma = 20°$.

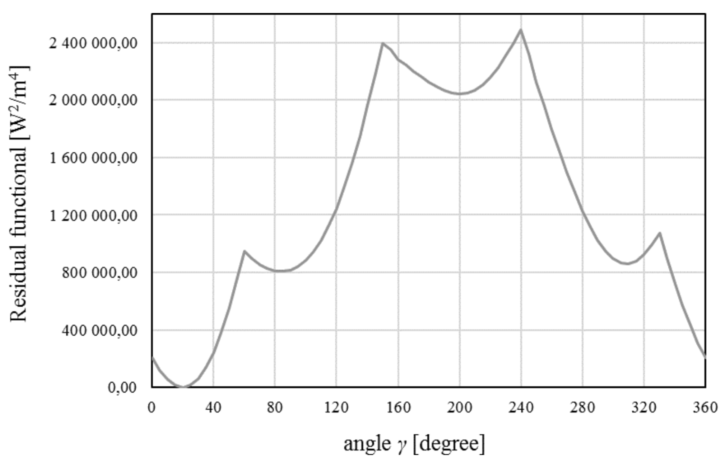


**Fig. 5.** Cross-section of the residual functional by the angle $\gamma$ and $\vartheta = 5°$, $\psi = 70°$.

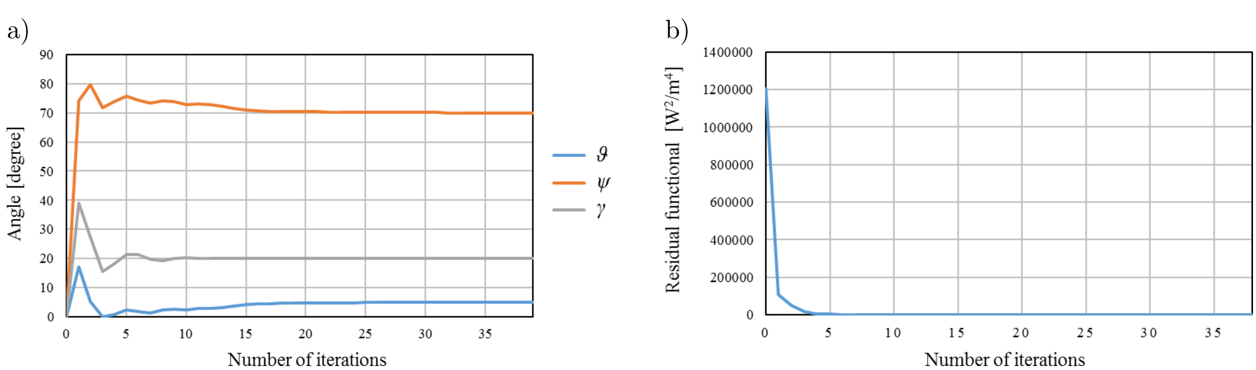


**Fig. 6.** a) Inverse problems solution for the "exact" values $\vartheta = 5°$, $\psi = 70°$, $\gamma = 20°$, b) the initial approximation for the conjugate gradient method is: $\vartheta = 0°$, $\psi = 0°$, $\gamma = 0°$.

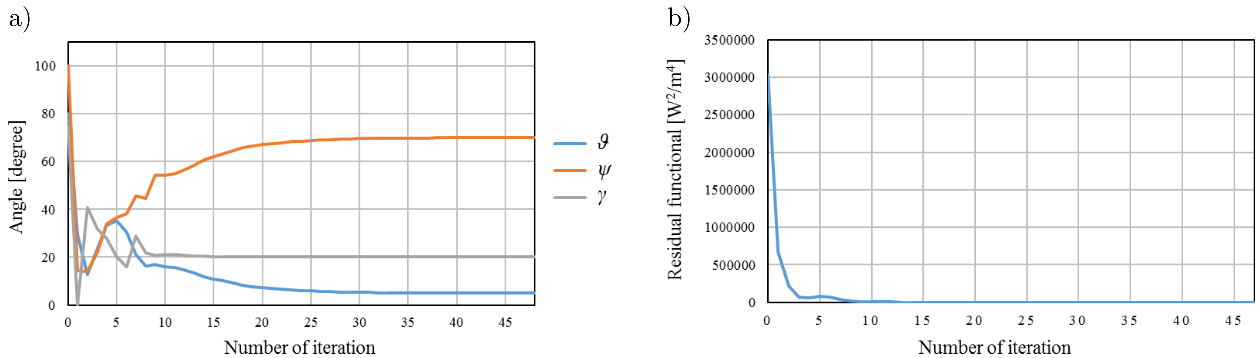


**Fig. 7.** a) Inverse problems solution for the "exact" values $\vartheta = 5°$, $\psi = 70°$, $\gamma = 20°$, b) the initial approximation for the conjugate gradient method is: $\vartheta = 80°$, $\psi = 100°$, $\gamma = 75°$.

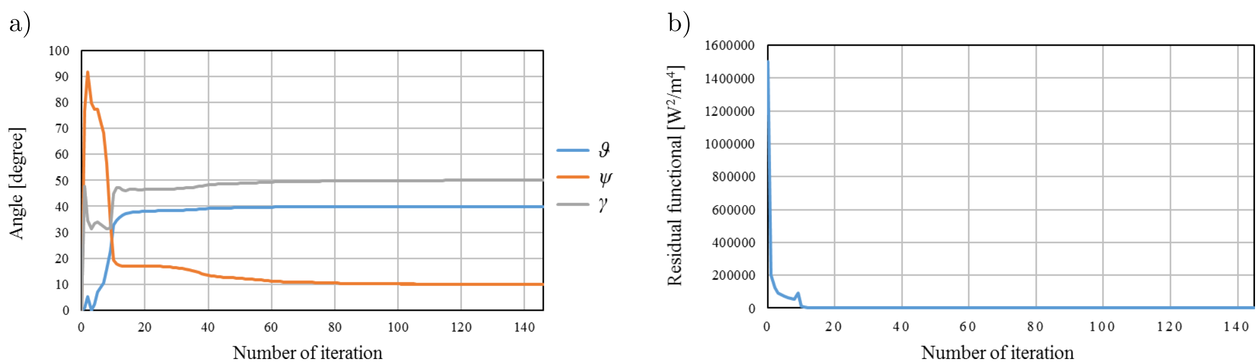


**Fig. 8.** a) Inverse problems solution results for the "exact" values $\vartheta = 40°$, $\psi = 10°$, $\gamma = 50°$, b) the initial approximation for the conjugate gradient method is: $\vartheta = 0°$, $\psi = 0°$, $\gamma = 0°$.

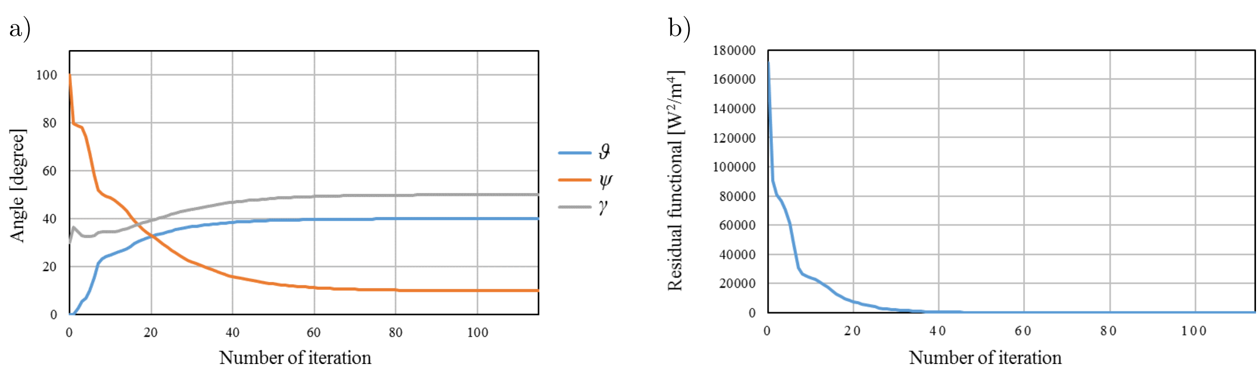


**Fig. 9.** a) Inverse problems solution results for the "exact" values $\vartheta = 40°$, $\psi = 10°$, $\gamma = 50°$, b) the initial approximation for the conjugate gradient method is: $\vartheta = 0°$, $\psi = 100°$, $\gamma = 30°$.

The number of restarts is selected by numerical simulation. In our case, 1000 restarts were selected.

At the end of the process, a vector of parameters $(\vartheta, \psi, \gamma)$ is chosen from all results of local optimization (34) at which the residual functional takes the smallest value. In this case, the residual functional agrees with the measurement error of the conjugate gradient method.

## 4. Results

The verification of the suggested algorithm was executed by the numerical simulation. At the beginning of the computational procedure, the thermoballistic direct problem for the parameters of orbit, the angular position of sensors and some arbitrary angles $\vartheta$, $\psi$ and $\gamma$ was solved. The calculated values of heat flux were used to simulate $q_m^{\rm exp}$, and then the values of $q_m^{\rm exp}$ were used to solve the inverse problem (23). Some results for the numerical simulation are presented in Figs 6–9.

In order to evaluate the accuracy of solving the geometric inverse problem, it is necessary to consider the effects of errors. The calculation was conducted on the effect of heat leaks from the heat flux sensors using the suggested method of numerical simulation.

Errors in the results of determining the angular orientation of the spacecraft depending on heat leakage are presented in Fig. 10. Moreover, both leakages from individual sensors and all six sensors were simulated simultaneously.

Heat leak is determined as

$$q_m = \overline{q}_m \, \delta q, \tag{46}$$

where $\overline{q}_m$ is the heat flux absorbed by the sensor, and $\delta q$ is the heat flux error.

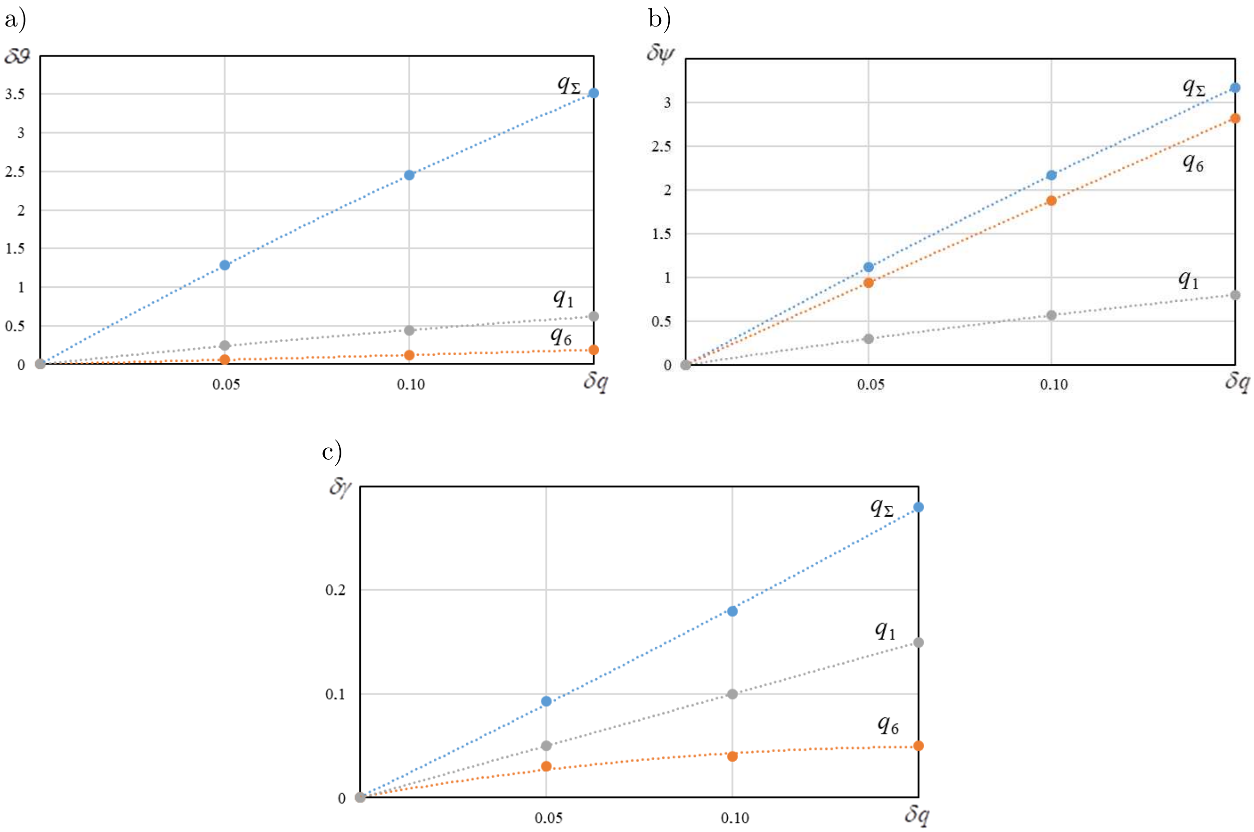


**Fig. 10.** Errors of recovery of the angular position of the spacecraft in the presence of heat leaks on the: a) 3rd, b) 6th and c) all six sensors simultaneously.

## 5. Conclusions

A generalized radiative transfer model, taking into account ballistic parameters and corresponding angles of orientation of spacecraft and its orbit to estimate the external heat fluxes, was developed. The rigorous theory of optimization was employed to solve the geometric inverse problem of estimating angles of orientation of spacecraft by measuring heat fluxes at the elements of the structure.

The computational results appear to be in good agreement with the simulated thermal measurements at conditions of spaceflight. This validation enables us to recommend the suggested approach for engineering estimates of the orientation of small satellites or as a backup system for spacecraft orientation. The results of the numerical simulation demonstrate sufficient numerical effectiveness of the suggested algorithm.

## Acknowledgments

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