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Identification of aerodynamic coefficients of a projectile and reconstruction of its trajectory from partial flight data

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Several optimization techniques are proposed both to identify the aerodynamic coefficients and to reconstruct the trajectory of a fin-stabilized projectile from partial flight data. A reduced ballistic model is used instead of a more general six degree of freedom (6DOF) ballistic model to represent the flight of the projectile. Optimization techniques are proposed in order to identify the set of aerodynamic coefficients. These techniques are compared when identifying the aerodynamic coefficients from both exact and noisy simulated partial flight data.

Keywords: aerodynamic coefficients, identification, free flight data, regularization.

1. Introduction

A projectile's inflight attitude is highly influenced by its geometry, which is represented in the ballistic model by aerodynamic coefficients. Accurate knowledge of the aerodynamic coefficients of a projectile is therefore essential in understanding, controlling and predicting the trajectory of a projectile. There are three main techniques that can be complementary in identifying the aerodynamic coefficients of a projectile: aerodynamic numerical codes [9], wind tunnel tests [2] and free-flight tests. The firing test, allowing real conditions to be reproduced, remains the most reliable method to study the aerodynamic behavior of a projectile. Our aim is to develop a technique to identify the aerodynamic coefficients using flight test measurements. Several studies on this topic have been conducted and most of them consist in minimizing the difference between the measured data and the calculated data. In the literature, one can find identification techniques of aerodynamic coefficients for various ballistic models: the linearized six degrees of freedom model [1, 9], the point mass model, the modified point mass model, etc. There are identification methods based on statistical approaches using a priori information [6, 7] (the maximum likelihood method), gradient methods [1, 7, 9] (Newton-Raphson, Levenberg-Marquardt) or measurement filtering (Kalman filters [7, 10]).

This paper is organized as follows: we first recall the forces and the moments acting on the projectile during its flight and the general formulation of a six degrees of freedom model. We then use a reduced model introduced by Demailly et al. [5] for the identification of the aerodynamic coefficients of a fin-stabilized projectile. This reduced model only takes into account the state parameters that are most representative of the motion of the fin-stabilized projectile, namely axial velocity v_i and roll rate ω_c. In Sec. 3, four identification procedures are presented. Finally, in the last section, all discussed identification techniques are compared to identify the aerodynamic coefficients and to reconstruct the trajectory of a projectile using partial and noisy data.

2. Reduced system of equations of motion

The identification of aerodynamic coefficients from flight data requires the use of a ballistic model which is representative of the inflight attitude.

2.1. General formulation

This study is devoted to the characterization of the aerodynamic coefficients of a fin-stabilized projectile. Consequently, some approximations can be made. Firstly, the aerodynamic coefficients are considered to be constant during the whole flight. Secondly, the wind velocity is assumed to be negligible with respect to the projectile velocity. Finally, the projectile has a tight trajectory and is considered to have a small angle of attack α ($\sin\alpha \sim \alpha$). Let us first introduce the notations used throughout the paper to represent the ballistic frames in which forces and moments will be written:

- $(\underline{i}, \underline{j}, \underline{k})$ is a fixed frame linked to the cannon where the unit vector $\underline{j}$ is the vertical vector directed upwards and linking the center of the earth and the cannon,
- $(\underline{t}, \underline{s}, \underline{h})$ is a mobile frame linked to the trajectory where the unit vector $\underline{t}$ is the unit velocity vector of the projectile,
- $(\underline{c}, \underline{a}, \underline{b})$ is a mobile frame linked to the projectile where the unit vector $\underline{c}$ is collinear with the longitudinal axis of the projectile.

The forces are expressed in the cannon reference frame $(\underline{i}, \underline{j}, \underline{k})$. Gravity force $\underline{g}$ and the Coriolis force $\underline{Cor}$ can be distinguished from aerodynamic forces (Fig. 1) such as the drag force $\underline{D}(C_x)$, the lift force $\underline{L}(C_z)$ and the Magnus force $\underline{K}(C_y)$ which are induced by the air flow around the projectile.

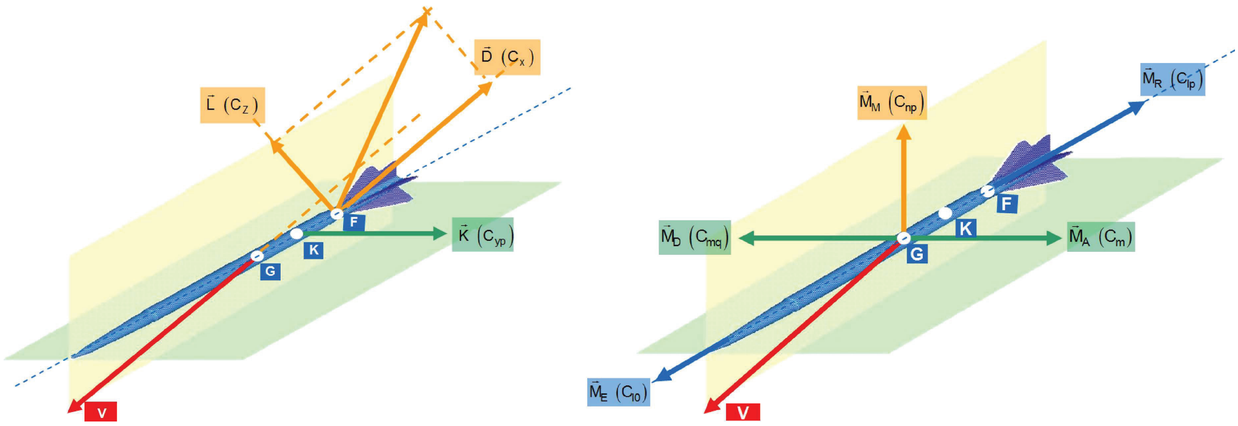

Fig. 1. Inventory of the forces (left), inventory of the moments (right) [5].

The drag force $\underline{D}$ acts on the projectile in the opposite direction of its velocity vector $\underline{v}$ and is located in the resistance plane formed by the vectors $\underline{t}$ and $\underline{c}$. The drag force is applied to the center of aerodynamic pressure (point F in Fig. 1). The angle of attack of the projectile (angle α, formed by $\underline{t}$ and $\underline{c}$) increases the section of the projectile in contact with the air stream and consequently increases the drag force $\underline{D}$:

$$\underline{D} = -qSC_x\underline{t}, \tag{1}$$

where q denotes the stagnation pressure and S the frontal area.

When the projectile is flying at a nonzero angle of attack, the distribution of pressure around the projectile becomes asymmetric and induces a force, called lift force $\underline{L}$:

$$\underline{L} = qSC_{z\alpha}[\underline{t} \wedge (\underline{c} \wedge \underline{t})], \tag{2}$$

which is normal for the velocity vector $\underline{v}$. This force is applied to the center of aerodynamic pressure (point F in Fig. 1).

When the projectile is in rotation around its longitudinal axis at a nonzero angle of attack, the force of friction generates an asymmetrical air flow around the projectile. This asymmetric flow induces a transverse force, called the Magnus force $\underline{K}$ that is applied at point K (Fig. 1):

$$\underline{K} = qSD\frac{\omega_c}{v}C_{yp\alpha}(\underline{c} \wedge \underline{t}), \tag{3}$$

where D is the projectile caliber.

The projectile is also subject to aerodynamic moments (Fig. 1):

- the rolling moment $\underline{M_E}(C_{l0})$:

$$\underline{M_E} = qSDC_{l0}\underline{c}; \tag{4}$$

- the roll damping moment $\underline{M_R}(C_{lp})$:

$$\underline{M_R} = -qSD^2\frac{\omega_c}{v}C_{lp}\underline{c}; \tag{5}$$

- the pitching moment $\underline{M_A}(C_m)$:

$$\underline{M_A} = qSDC_{m\alpha}(\underline{t} \wedge \underline{c}); \tag{6}$$

- the pitch damping moment $\underline{M_D}(C_{mq})$:

$$\underline{M_D} = \frac{-qSD^2}{v}C_{mq}(\underline{c} \wedge \underline{\dot{c}}); \tag{7}$$

- the Magnus moment $\underline{M_M}(C_{np})$:

$$\underline{M_M} = -qSD^2\frac{\omega_c}{v}C_{np\alpha}[\underline{c} \wedge (\underline{c} \wedge \underline{t})]. \tag{8}$$

The fundamental principle of dynamics leads to a system (6DOF model) of first-order nonlinear differential equations which gives the time evolution of the position and of the angular attitude of the projectile:

$$\begin{bmatrix} \dot{v_i} \\ \dot{v_j} \\ \dot{v_k} \end{bmatrix}_{(\underline{i},\underline{j},\underline{k})} = \frac{1}{m}(\underline{D} + \underline{L} + \underline{K}) + \underline{Cor} + \underline{g}, \tag{9}$$

$$\begin{bmatrix} I_1\dot{\omega_c} \\ I_2\dot{\omega_a} + \omega_c\omega_b(I_1 - I_2) \\ I_2\dot{\omega_b} + \omega_c\omega_a(I_2 - I_1) \end{bmatrix}_{(\underline{c},\underline{a},\underline{b})} = \underline{M_E} + \underline{M_R} + \underline{M_A} + \underline{M_D} + \underline{M_M}\ . \tag{10}$$

For the sake of simplicity, the state parameters are denoted by $U = (v_i, v_j, v_k, \omega_c, \omega_a, \omega_b)$, the aerodynamic coefficients are denoted by $C = (C_x, C_{z\alpha}, C_{yp\alpha}, C_{l0}, C_{lp}, C_{m\alpha}, C_{mq}, C_{np\alpha})$ and the dynamical system corresponding to the fundamental principle of dynamics is denoted in simplified form:

$$\dot{U} = f(U, C, t). \tag{11}$$

2.2. Reduced ballistic model for a fin-stabilized projectile

For a fin-stabilized projectile, one can find in the literature several closed form solutions of the projectile trajectory, which are based on different assumptions. In particular, it has been shown that equations governing the evolution of v_i and ω_c can be decoupled from other equations [1, 5]. Demailly et al. [5] realized a sensitivity analysis with respect to each aerodynamic coefficient showing that the axial velocity parameter v_i only depends on the drag coefficient C_x and that the roll rate parameter ω_c is only influenced by the coefficients C_x, C_{lp} and C_{l0}. Consequently, only the drag force $\underline{D}$, the rolling moment $\underline{M_E}$ and the roll damping moment $\underline{M_R}$ can be considered in the reduced ballistic model of a fin-stabilized projectile. This leads to the following equations:

$$\begin{cases} \dot{v}_i = -\dfrac{\rho S}{2m} v_i^2 C_x, \\[2ex] \dot{\omega}_c = -\dfrac{\rho S}{2I_1}[D^2 C_{lp}\omega_c v_i - DC_{l0} v_i^2]. \end{cases} \tag{12}$$

The simplified ballistic model for a fin-stabilized projectile (12) is then denoted in simplified form

$$\dot{U} = g(U, C, t), \tag{13}$$

where the vector $U = (v_i, \omega_c)$ contains the state parameters and $C = (C_x, C_{lp}, C_{l0})$ is a vector containing the aerodynamic coefficients.

It can be noticed that the differential system (12) can be analytically solved if the aerodynamic coefficients C are assumed to be constant. The analytical solution is given by

$$v_i(t) = \frac{v_{i_0}}{1 + \dfrac{\rho S C_x v_{i_0}}{2m} t}, \tag*{$(14)_1$}$$

$$\omega_c(t) = (\omega_{c_0} + v_{i_0}\gamma)\left(\frac{v_i(t)}{v_{i_0}}\right)^{\beta} - v_i(t)\gamma, \tag*{$(14)_2$}$$

where

$$\gamma = \frac{mDC_{l0}}{I_1 C_x - mD^2 C_{lp}}, \qquad \beta = \frac{mD^2}{I_1}\frac{C_{lp}}{C_x}. \tag{15}$$

Thus, the solution involves five parameters which are the three aerodynamic coefficients C_x, C_{lp}, C_{l0} and the two initial conditions v_{i_0}, ω_{c_0}.

3. Nonlinear identification procedures of aerodynamic coefficients

During a flight, discrete and partial measurements of the state parameters v_i and ω_c are recorded. Let $\Omega \subset \mathbb{R}^+$ be the time range of the flight. We define a partition of $\Omega = \Omega_d \cup \Omega_i$, where Ω_d denotes the periods in which the measures ϕ_d of the flight are available. Let ϕ be the linear interpolated extension of ϕ_d on the whole time range Ω. The principle of the identification method is to look for the set of aerodynamic coefficients (C_x, C_{lp} and C_{l0}) associated with the couple (v_i^*, ω_c^*) that are as close as possible to the measurements $\phi_d = (\widetilde{v_i}, \widetilde{\omega_c})$ and satisfying (as much as possible or exactly) the flight mechanics equations:

$$\dot{U} = g(U, C, t) \quad \forall t \in \Omega. \tag{16}$$

3.1. Penalization method (PM)

Demailly et al. [5] proposed a nonlinear optimization problem which minimizes the distance between the measured data and a calculated flight using the least squares method

$$\begin{cases} \text{Find} \;\; \Psi = (U^*, C^*) \text{ such as} \\ J(\Psi^*) \leq J(\Psi) \;\; \forall\, \Psi = (U, C), \\ J(\Psi) = ||U - \phi||^2_\Omega + \eta ||R(\Psi)||^2_\Omega. \end{cases} \tag{17}$$

A penalization term $||R(\Psi)||_\Omega$ is introduced to take into account the flight mechanics equations (Eq. (16)). We then have to discretize the problem in order to use real or numerical data. The penalization term, if we use, for example, the Euler explicit integration scheme becomes:

$$||R(\Psi)||^2_\Omega = \sum_{m=1}^{N-1} [U^{m+1} - U^m - f(U^m, C, t^m)(t^{m+1} - t^m)]^2.$$

The Newton-Raphson technique is then used to solve the system of non-linear optimality equations. The disadvantage of using a penalization term is that the solution of the optimization problem depends on the choice of the parameter η.

3.2. Constrained optimization method (COM)

To fully take into account the flight mechanics, the authors propose another method that uses equality constraints, which leads to the following constrained minimization problem:

$$\begin{cases} \text{Find} \;\; \Psi^* = (U^*, C^*) \text{ which minimizes} \\ J(\Psi) = ||U - \phi||^2_\Omega \;\; \forall\, \Psi = (U, C), \\ \text{under the equality constraints (16).} \end{cases} \tag{18}$$

In this situation, the matrix system size $(4N \times 4N)$ is greater compared to the use of a penalization term in the functional $((2N+2) \times (2N+2))$. This is due to the use of Lagrange multipliers in order to take into account the equality constraints. The calculation time is consequently increased.

We can notice that both methods do not discriminate between the measured data ϕ_d and the interpolated data ϕ. In other words, these methods minimize the distance not only to the measured data, but also to the interpolated data.

3.3. Fading regularization method (FRM)

Measurements provided by instrumentation are not available on the entire flight. We introduce a new method, the fading regularization method (FRM), in order to simultaneously identify the aerodynamic coefficients and reconstruct the entire flight. The method is inspired by approach presented in [3, 4] introduced by Cimetière et al. It consists of a sequence of nonlinear constrained optimization problems. The functional J^k contains two terms. The first term characterizes the validity given to measurements ϕ_d and the second one characterizes the distance to the solution obtained at the previous iteration U^k. This is a regularization term which allows the optimization problem to be well-posed.

$$\begin{cases} \text{Find} \;\; \Psi^{k+1} = (U^{k+1}, C^{k+1}) \text{ which minimizes} \\ J^k_c(\psi) = ||U - \phi_d||^2_{\Omega_d} + c||U - U^k||^2_\Omega \quad \text{with } \psi = (U, C), \\ U^0 = \phi, \\ \text{under the equality constraints (16).} \end{cases} \tag{19}$$

This sequence of optimization problems converges to $\Psi^n = (U^n, C^n)$ where U^n is the entire reconstructed trajectory that is the closest to the measurements ϕ_d and C^n is the identified aerodynamic coefficients vector. It is not proven here, but the solution is supposed to be independent both of the regularization parameter c and the initialization of the state parameters U^0.

3.4. Least squares method using the analytical solution (least squares)

When the aerodynamic coefficients are constant, the reduced ballistic model has an analytic solution and we can use the classical least squares identification method to obtain the parameters.

The identification procedure is divided into two steps. The first one is an optimization problem that identifies C_x^* and $v_{i_0}^*$. We minimize J_1, which represents the distance between the measured data $\widetilde{v_i}$ and the calculated data $v_i(t)$ taking into account the first equation of the analytical solution $(14)_1$

$$\begin{cases} \text{Find} \quad (C_x^*, v_{i_0}^*) \text{ which minimizes} \\ J_1(C_x, v_{i_0}) = \|v_i(t) - \widetilde{v_i}\|^2_{\Omega_d}. \end{cases} \tag{20}$$

The second step is an optimization problem that identifies C_{lp}^*, C_{l0}^* and $\omega_{c_0}^*$, considering the solutions $C_x^*, v_{i_0}^*$ found at the first step. We minimize J_2, representing the distance between the measured data $\widetilde{\omega_c}$ and the calculated data $\omega_c(t)$ taking into account the second equation of the analytical solution $(14)_2$.

$$\begin{cases} \text{Find} \quad (C_{l0}^*, C_{lp}^*, \omega_{c_0}^*) \text{ which minimizes} \\ J_2(C_{lp}, C_{l0}, \omega_{c_0}) = \|\omega_c(t) - \widetilde{\omega_c}\|^2_{\Omega_d}, \\ \text{such as} \quad C_x = C_x^* \quad \text{and} \quad v_{i_0} = v_{i_0}^*. \end{cases} \tag{21}$$

In both cases, the optimality equations are nonlinear and the Newton-Raphson technique is then used to solve the nonlinear system. This least squares identification method gives us a reference and allows us to compare the efficiency and the accuracy of the different identification techniques we proposed.

4. Numerical results

The efficiency of the above identification techniques is illustrated in different test situations. Measurements for axial velocity v_i and roll rate ω_c are numerically generated using a time integration of the reduced ballistic model $(14)_1$–$(14)_2$. In real situations, the roll rate measurements are provided by two different intrumentations: yaw cards positioned at the beginning of the flight and a radar reflector at the end of the trajectory. There is therefore a gap where no measurements are available between the yaw cards (beginning of the flight) and the radar (end of the flight).

4.1. First scenario: identifications using simulated non-noised data

Figure 2 gives the reconstructions of the roll rate parameter ω_c using the different identification techniques from simulated yaw cards data and radar data. For the sake of confidentiality, curves have been normalized ($\omega_c^* = \omega_c/\omega_{c_{\max}}$ and $t^* = t/t_{\max}$). In addition, Table 1 gives the relative errors made on the identified coefficients and initial conditions for each identification technique. Figure 2 and Table 1 show that the FRM gives the best results. Indeed, this technique is able to take into account the lack of data, to simultaneously identify with precision the aerodynamic coefficients and the initial conditions v_{i_0} and ω_{c_0} and to reconstruct the whole trajectory of the projectile.

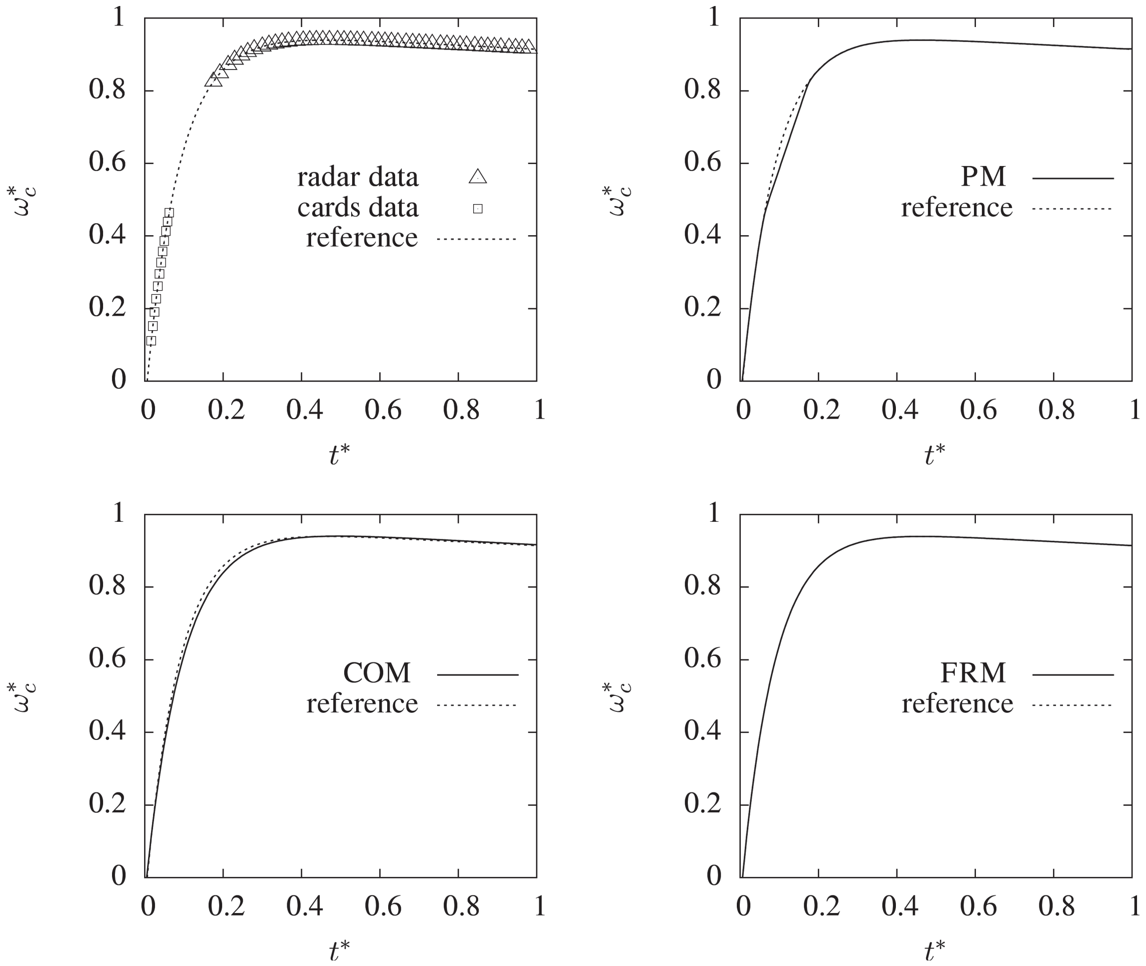

Fig. 2. Reconstruction of the roll rate parameter ω_c using simulated non-noised yaw cards and radar data.

Table 1. Relative errors using simulated non-noised yaw cards and radar data.

	Relative error (%)					CPU Time [s]
	C_x	C_{lp}	C_{l0}	v_{i_0}	ω_{c_0}	
PM	0.15	11.02	10.75	X	X	0.6
COM	3.72	9.18	8.93	0.12	29.44	6
FRM	$1.7.10^{-3}$	5.10^{-3}	$4.9.10^{-3}$	5.10^{-4}	2.22	6

4.2. Second scenario: identifications using simulated noisy data

Figure 3 gives the reconstructions of the roll rate parameter ω_c using the different identification techniques from simulated noisy data. The noise level on the axial velocity parameter is set to 0.1% and the noise level on the roll rate parameter is set to 5%. Table 2 gives the relative errors made on the identified coefficients and initial conditions for each identification procedure. Figure 3 and Table 2 prove the robustness of the FRM when dealing with noisy data. Indeed, Fig. 3 shows that the solution given by the FRM is denoised. In addition, this identification technique gives results that are almost as accurate as the ones obtained by the classical least squares identification method using an analytical solution.

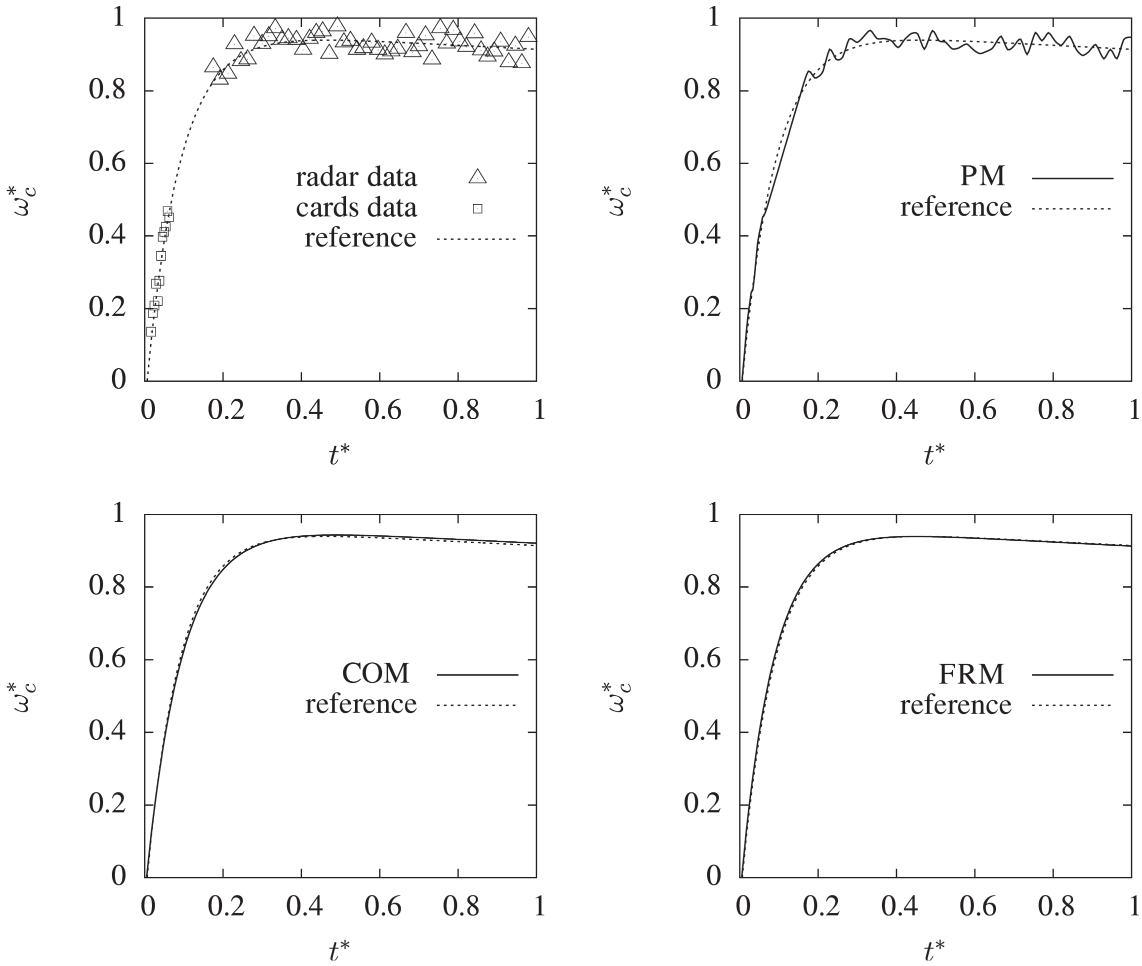

Fig. 3. Reconstruction of the roll rate parameter ω_c using simulated noisy yaw cards data and radar data.

Table 2. Relative errors using simulated noisy yaw cards data and radar data.

	Relative error (%)					CPU Time [s]
	C_x	C_{lp}	C_{l0}	v_{i_0}	ω_{c_0}	
PM	5.1	6.03	5.33	X	X	1
COM	3.89	6	5.43	0.13	28.4	9
FRM	1.82	2.43	2.33	0.05	20.48	10
Least Squares	1.78	2.43	2.33	0.05	18.17	2

4.3. Third scenario: identifications with only radar data

In this scenario we want to know if we are able to reconstruct the beginning of the trajectory from radar data that is only available at the end of the trajectory. Figure 4 gives some reconstructions at different steps of the iterative process using the FRM. The corresponding identified aerodynamic coefficients are presented in Table 3. In order to make comparisons, the other identification techniques are used with the available data on the measured part of the trajectory Ω_d. The obtained results are given in Table 3.

FRM allows not only the aerodynamic coefficients to be identified with better accuracy than the other methods, but also allows the initial conditions to be precisely identified. It also shows that it

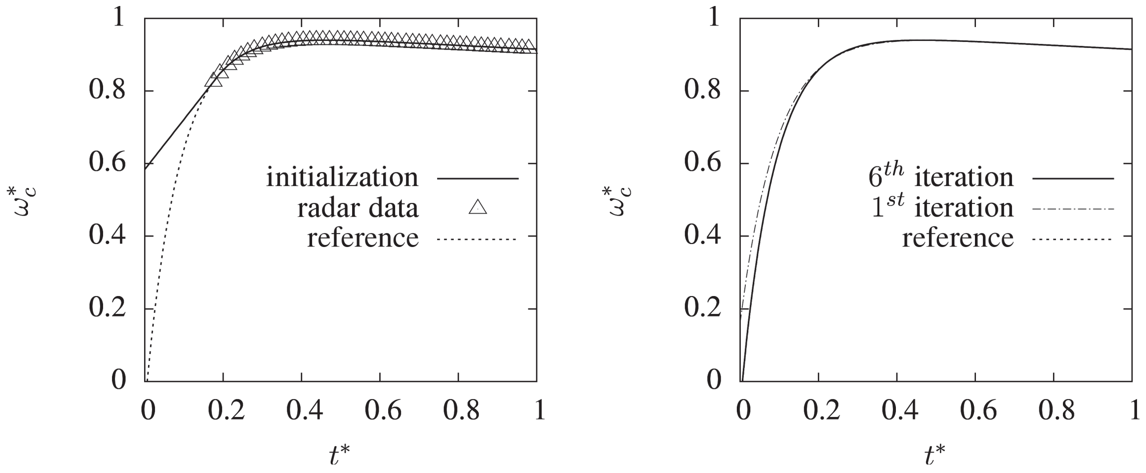

Fig. 4. Reconstruction of the roll rate parameter ω_c using only radar data.

Table 3. Relative errors using only radar data.

	Relative error (%)					CPU
	C_x	C_{lp}	C_{l0}	v_{i_0}	ω_{c_0}	Time [s]
PM on Ω_d	0.03	4.79	4.69	X	X	1
COM on Ω_d	1.3	1	1	X	X	8
FRM on Ω	0.01	0.19	0.19	4.10^{-4}	9	30

is possible to identify the aerodynamic coefficients and initial conditions of the flight without yaw cards data at the beginning of the trajectory.

5. Conclusion

Different identification techniques of the aerodynamic coefficients of a fin-stabilized projectile from inflight data were proposed. A reduced ballistic model proposed by Demailly et al. [5] was used to represent the flight. Some numerical simulations using simulated data have shown that the fading regularization method gives the best identification results. Moreover, when there is a lack of flight data on one part of the trajectory, FRM enables to simultaneously and accurately identify the aerodynamic coefficients and the initial conditions and to reconstruct the whole trajectory of the projectile. The proposed technique will be used in further works with real flight data and with ballistic models where no analytical solution is available.

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