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Self-learning FEM/NMM approach to identification of equivalent material models for plane stress problem

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The autoprogressive and cumulative algorithms, basing on 'on line' formulation of patterns and the training of NMM (Neural Material Model), are evaluated to be comparable in case of uniaxial stress state problems. It is shown in the paper that for the plane stress boundary value problems the autoprogressive algorithm, in which NMM is trained for each load increment, is superior to the cumulative algorithm. In order to formulate a small NMM and accelerate the convergence of the iteration of computed equilibrium paths to the monitored paths, a smaller number of inputs NMM is discussed and a modified selection of the training patterns is applied. A new approach is proposed with respect to the designing of NMMs, combining the 'on line' and 'off line' training of neural networks. The discussed problems are illustrated with two study cases. They are related to the formulation of NMMs for the identification of equivalent materials in plane trusses made of the Ramberg–Osgood material and for elasto-plastic plane stress boundary value problems.

Keywords: neural networks; material model; constitutive modelling

1. Introduction

Neural Networks (NN) have been efficiently used for modelling of various materials. The first research on neural network based material model (NMM) was originated by J. Ghaboussi research team at Univ. of Illinois in 1991. In paper [2] simple NMM models were formulated for ordinary concretes on the base of laboratory tests for the plane stress state. Paper [2] originated implicit modelling of materials [1] using NNs as 'material free, data dependent' tools.

From among many papers devoted to formulations of NMMs paper [24] is worth mentioning since it discussed Japanese results associated with modelling of constitutive equations of rate-dependent models (viscoplastic models) of steel and other metals alloys subjected to cyclic loadings. The main goal of NMM modelling was to obtain NNs which could be used in hybrid FEM/NMM programs instead of numerically inefficient standard procedures.

In [21] this approach was used for modelling the Return Mapping Algorithm (RMA) which was formulated in mid of 1980's to compute a consistent stiffness matrix of elastoplastic materials. In this paper a great number of pseudo-experimental patterns was computed by means of a numerical procedure and then the trained NMM was used in a FEM/NMM program to the analysis of plane-stress boundary-value problems. A generalised RMA was analysed in [10, 22] where the computer simulated patterns took into account the bending plate internal constraint hypotheses.

The above discussed approach was based on the 'off line' technique of the NMM design using patterns from tests on laboratory models or pseudo-experimental, i.e. computer simulations. The approach opens the door for the analysis of problems with complex models of materials, cf. e.g. analysis of soil materials [4, 17] and geotechnics problems [7], composite materials [11] and cryogenic composite cables [12, 13], as well as multiscale modelling of materials [5].

The main problem of the 'off line' approach is the pattern generation, general enough for formulation of NMMs which could be used in FEM/NMM programs for the analysis of a variety of boundary value problems.

Another approach was proposed in the pioneering work [3]. The main idea was to generate patterns for the 'on line' training of NMM inside a FEM/NMM program. In this method NMM is formulated iteratively during two-stage computation at each load increment. It enables modification of patterns gradually 'calibrating' them by means of structural responses monitored in selected control points. This approach, called in [3] the autoprogressive training method, makes it possible to formulate an equivalent material model (in the sense of classical homogenisation theory [19]) exploring neural network implicit modelling possibilities. The autoprogressive algorithm was developed in [6] and has recently been applied to the analysis of rate-dependent materials [8, 9].

As mentioned above, in the autoprogressive method both the patterns and NMMs are computed and retrained at each load increment. A slightly different approach was proposed by Pande *et al.* from Swansea College, UK [15]. Patterns are generated at each load increment but NMM is trained after the ending of the loading process. Both approaches were carefully analysed in [14] (cf. also [20]) for uniaxial stresses in structures (following [3] a plane truss was analysed as a bench-mark example). It was stated that both applied algorithms, i.e. the autoprogressive and cumulative algorithms have similar numerical efficiency.

More complicated identification problems were discussed in [6, 16] where multiaxial stress-strain relations were analysed. In these papers parametric identification of constitutive matrix for anisotropic materials was proceeded exploring analytical relations between input/output of NMM computed for a linear elastic material model. The autoprogressive algorithm was also applied in [6] to the implicit identification of NMM for a plane stress linear elastic-plastic material but it was stated that the convergence of the algorithm was worse than in the case of uniaxial stress state.

The question of convergence of the 'on line' technique algorithms is discussed in the present paper. The introductory formulation of networks and the way of selection of generated 'on line' patterns are considered as crucial questions strongly influencing the final NMMs due to their numerical efficiency related to quick convergence of identification algorithms. An important question on the generalisation properties of the identified NMMs, initiated in [8, 9] for the uniaxial stress state, is developed in the present paper with respect to the plane stress boundary value problems of elastoplastic materials models.

2. AUTOPROGRESSIVE TRAINING OF NMM

2.1. Some general remarks on NMM

Two approaches to the 'on line' training of NMM, i.e. the autoprogressive and cumulative methods were discussed in [14]. A final conclusion that both algorithms are similar to each other was formulated in this paper on the base of uniaxial stress state analysis. This conclusion was not valid in the analysis of plane stress boundary-value problem.

In [23] a short report from research, conducted as present, on the identification of elastoplastic material in the plane stress state, only results by the autoprogressive algorithm were presented. In this analysis the autoprogressive algorithm was demonstrated to be more efficient than the cumulative algorithm.

Following papers [3, 6] the history of loading process was taken into account using time-delay input data to the training of NMM. For instance, including only one back-time instant n with respect to the output instant $n+1$ gives the following input and output vectors components,

$$\boldsymbol{x}_{(9\times1)} = \{{}_{n+1}\boldsymbol{\varepsilon},\ {}_{n}\boldsymbol{\varepsilon},\ {}_{n}\boldsymbol{\sigma}\}, \qquad \boldsymbol{y}_{(3x1)} = \{{}_{n+1}\boldsymbol{\sigma}\}, \tag{1}$$

where

$${}_{k}\boldsymbol{\varepsilon} = {}_{k}\{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy}\}, \qquad {}_{k}\boldsymbol{\sigma} = {}_{k}\{\sigma_x,\ \sigma_y,\ \tau_{xy}\} \qquad \text{for} \quad k = n, n+1. \tag{2}$$

This leads to the $9 \times H1 \times H2 \times 3$ multi-layer NN architecture, where two hidden layers with $H1$ and $H2$ sigmoidal neurons were applied. The number of input and output neurons is fixed, whereas the number of hidden layers neurons depends on the analysed problem in question.

It was verified numerically, cf. [20], that the part of input vector (1), i.e. ${}_n\boldsymbol{\varepsilon}$, ${}_n\boldsymbol{\sigma}$ for the instant n can be eliminated, and now the vector is of the form

$$\boldsymbol{x}_{(3\times1)} = \{{}_{n+1}\boldsymbol{\varepsilon}\} = {}_{n+1}\{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy}\}. \tag{3}$$

This enables a reduction of the network to smaller architecture: $3 \times H1 \times H2 \times 3$. The number of input and output neurons is fixed, whereas the number of hidden layers neurons depends on the analysed problem. Now it is possible to compute the incremental constitutive relations over given strain increment, which are a consistent Jacobian

$${}_{n+1}\boldsymbol{J} = \frac{\partial_{n+1}\Delta\boldsymbol{\sigma}}{\partial_{n+1}\Delta\boldsymbol{\varepsilon}}, \tag{4}$$

where

$${}_{n+1}\Delta\boldsymbol{\sigma} = {}_{n+1}\boldsymbol{\sigma} - {}_n\boldsymbol{\sigma}, \qquad {}_{n+1}\Delta\boldsymbol{\varepsilon} = {}_{n+1}\boldsymbol{\varepsilon} - {}_n\boldsymbol{\varepsilon}. \tag{5}$$

The components of the Jacobian (4) refer to the consistent tangent stiffness matrix, cf. [18]. In paper [6] the analytical formulae were derived for the calculation of partial derivatives (4).

They depend on the given input and output values and on the NMM parameters. This means that the trained NMM is not 'a black box' but in fact it consists of a relationship of stress to strain and can be used for computing the material constitutive matrix.

2.2. Algorithm of the autoprogressive method

The autoprogressive method of NMM training was extensively discussed in [14] vs. the cumulative method, basing on a plane truss example. In the present paper we apply only the autoprogressive method (Algorithm A) from [14] so in Fig. 1 a shortened flow chart of the corresponding algorithm is drawn. The two stages used in the flow chart are illustrated on an example of a plane stress beam shown in Fig. 2.

The flow chart in Fig. 1 is referred to the incremental form of the finite element method (FEM). A single load parameter λ is assumed as a multiplier of the reference load vector $\mathbf{P}^\star$, $(\mathbf{P} = \lambda\,\mathbf{P}^\star)$. The loading process is controlled by the λ parameter applying its increments $\Delta_n\lambda$ for $n = 1, 2, \ldots, NP$. These increments are unchanged for each loading cycle c. Two stage computations are performed at the load level n. At Stage I, displacement fields are computed by a FEM, using a current NN material model with the given load. Stage II serves to the compensation of measured and computed deformations if they appear, of course.

Stage I

This stage fully corresponds to the Newton–Raphson incremental algorithm. To compute displacements ${}_nu_j$ at the measurement points j and at each increment $\Delta_n\lambda$, the hybrid FEM/${}^c_n$NMM program is used. Then the strains and stresses vectors ${}^{\mathrm{I}}\boldsymbol{\varepsilon}^g$, ${}^{\mathrm{I}}\boldsymbol{\sigma}^g$ are computed and stored. At this stage a finite element analysis is applied using the ${}^c_{n-1}$NN material model obtained at Stage II for the previous load step $n-1$.

Stage II

This stage is realised when displacement criterion is fulfilled:

$${}_nd_j \equiv {}_n|u^m_j - u_j| / \max(|{}_nu|) \cdot 100\% > \mathrm{er}u_{\mathrm{adm}}\,, \tag{6}$$

where: ${}_nu$ – computed displacements for load step n, ${}_nu^m_j$ – measured displacements, $\mathrm{er}u_{\mathrm{adm}}$ – admissible error. The extorted difference of displacements ${}_nd_j$ is used to compute the strains and stresses vectors $\{{}^{\mathrm{II}}\boldsymbol{\varepsilon},\ {}^{\mathrm{II}}\boldsymbol{\sigma}\}$ applying the same ${}^c_{n-1}$NMM model as at Stage I of the FEM analysis.

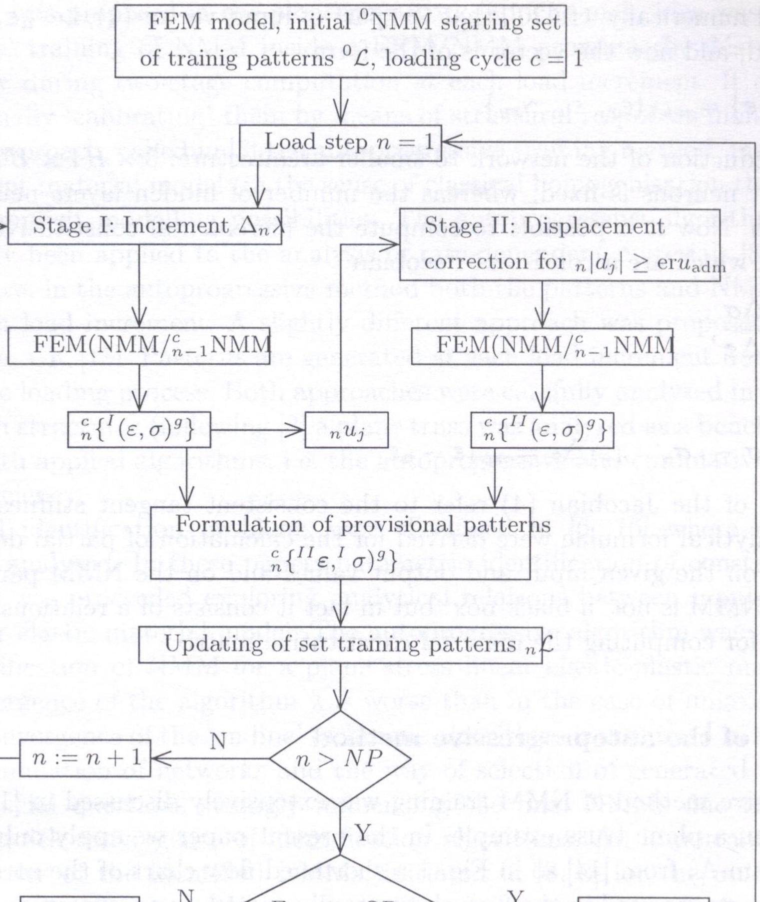


Fig. 1. Flow chart of autoprogressive algorithm

Provisional and fixed patterns

Two types of the patterns sets are composed of pairs $\{\boldsymbol{\varepsilon}^g,\ \boldsymbol{\sigma}^g\}_{g=1}^{Ng}$ chosen from the Gauss integration points g of the finite elements e for Stages I and II, cf. Fig. 2. When the difference between displacements u_j computed at Stage I and the measured displacements u_j^m to be found, cf. Eq. (6) the provisional patterns are formulated ${}^c_n\{{}^{\mathrm{II}}\boldsymbol{\varepsilon}^g,\ {}^{\mathrm{I}}\boldsymbol{\sigma}^g\}$ to updating a set of training patterns

$$\mathcal{L} = \{(\boldsymbol{\varepsilon},\ \boldsymbol{\sigma})^p\}_{p=1}^{L}. \tag{7}$$

Otherwise, i.e. if the error criterion (6) for a certain $n^\star$ load step is satisfied, the fixed patterns ${}^c_{n^\star}\{{}^{\mathrm{I}}\boldsymbol{\varepsilon}^g,\ {}^{\mathrm{I}}\boldsymbol{\sigma}^g\}$ are used to updating the set (7).

The provisional patterns are accumulated during the processing along the loading cycle c. After this process these patterns are subsequently changed at the next cycle $c+1$. The fixed patterns set is extended only and it is not subject to change. The training set of patterns should be completed very carefully. All the carried out numerical experiments show that the number of pairs $\{\boldsymbol{\varepsilon},\ \boldsymbol{\sigma}\}^p$ and the placing of FEs from which they are taken have an essential meaning for the computational efficiency. This concerns the number of load cycles needed to the correct identification of NMM model.

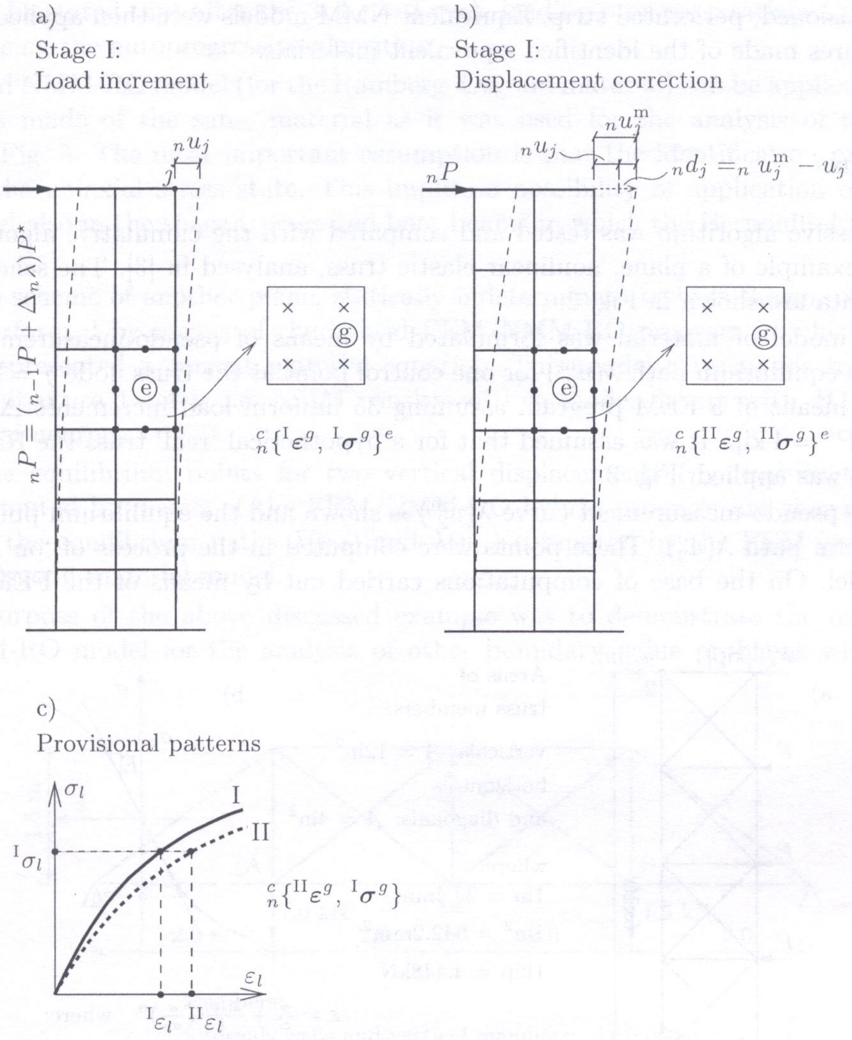


Fig. 2. Stress and strains for stages I, II and provisional patterns formulation in subsequent Gaussian integration points $g = 1, \ldots, N_g$

Training of NN material model

According to the algorithm shown in Fig. 1, the training of NN should start from an initial material model. This model, which usually represents a linear elastic material behaviour, can be prepared for instance on the base of a linear constitutive equation.

In the present paper another way was proposed for the generation of the initial data sets. These sets were computed by a FEM program in the analysis of the adopted boundary value problem assuming the initial linear elastic material. The strains and stresses from Stages I and II serve to formulate the initial data set $\mathcal{L} =^0 \{^{\text{II}}\varepsilon, {}^{\text{I}}\sigma\}$. This set was used for the 'off line' training of the initial ^{0}NMM. The neural material model was then updated in subsequent cycles $c = 1, 2, \ldots, CS$ up to the final model which well represents the real material behaviour.

The identified material ^{CS}NMM, obtained by means of autoprogressive training, can be applied to the analysis of other boundary value problems corresponding to the same equivalent material.

3. Numerical examples

Below the identification of NMM models is discussed for two study cases. The first case deals with the bench-mark type plane truss, analysed in [3]. The other study case is related to plane stress elastoplastic tensioned, perforated strip. Equivalent NMM models were then applied to the analysis of other structures made of the identified equivalent materials.

3.1. Plane truss

The autoprogressive algorithm was tested and compared with the cumulative algorithm in [14] on a bench-mark example of a plane, nonlinear elastic truss, analysed in [3]. The scheme of the truss and material data are shown in Fig. 3.

The neural model of material was formulated by means of pseudo-measurement data corresponding to the equilibrium path $\lambda(u_2^{\mathrm{m}})$ for one control point at the truss node $j = 2$. The path was determined by means of a FEM program assuming 35 uniform load increments $\Delta\lambda = 0.5$ for the reference load $P^\star = 1\,\mathrm{kip}$. It was assumed that for a hypothetical 'real' truss the Ramberg–Osgood material model was applied, Fig. 3.

In Fig. 4 the pseudo-measurement curve $\lambda(u_2^{\mathrm{m}})$ is shown and the equilibrium points at evolution of the equilibrium path $\lambda(u_2^{\mathrm{c}})$. These points were computed in the process of 'on line' training of the NMM model. On the base of computations carried out by means of the FEM/NMM hybrid

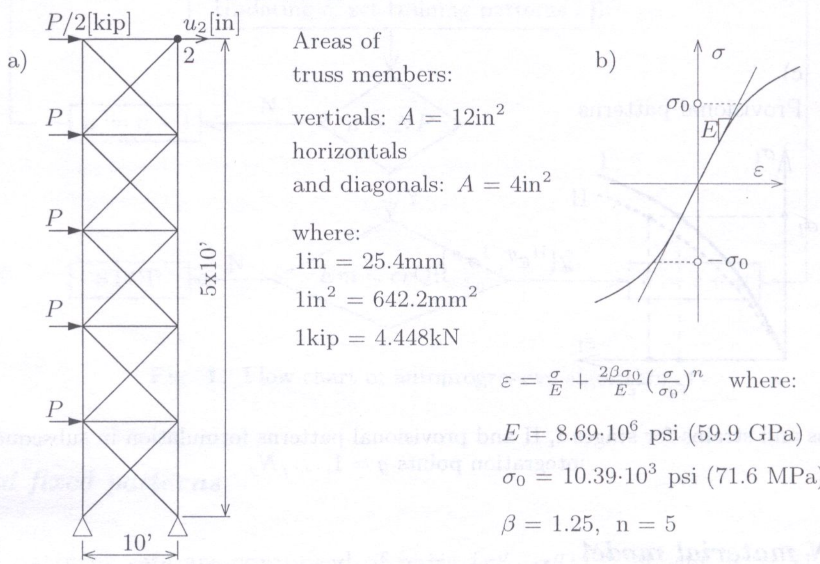


Fig. 3. Data of truss structure from [3].

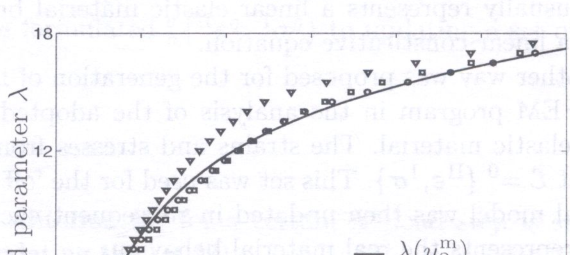


Fig. 4. Pseudo-measurement curve $\lambda(u_2^{\mathrm{m}})$ of horizontal displacement of truss node $j = 2$, and load paths corresponding to autoprogressive algorithm for $c = 3$ load cycles.

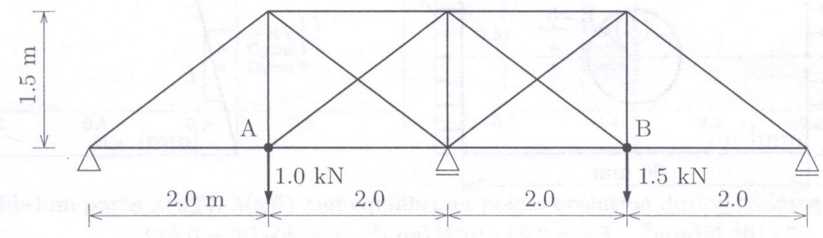
program, it can be stated that after the 3rd $c = 3$ cycle loading process, a satisfactory NMM model was identified using the autoprogressive algorithm.

The identified NMM-RO model (for the Ramberg–Osgood material) can be applied to the analysis of other trusses made of the same material as it was used for the analysis of the bench-mark truss shown in Fig. 3. The most important assumption is that the identification process has been performed for the uniaxial stress state. This implies a possibility of application of the identified NMM-RO model also in the space trusses and bent beams in which the Bernoulli–Euler hypotheses apply.

In Fig. 5 the scheme of another plane, statically indeterminate truss is shown. Now the computations were performed by means of the hybrid FEM/NMM-RO program in which the identified neural model represented a current material equation. This model corresponds to the NMM-RO network of architecture 1-4-4-1, i.e. NMM consists of two hidden layers with $H1 = H2 = 4$ of bipolar sigmoidal neurons.

In Fig. 6 the equilibrium points for two vertical displacements (v_A, v_B) at the truss nodes ($j = A, B$) computed by means of the FEM/NMM-RO hybrid program are shown. Those points are situated at the equilibrium paths $\lambda(v_A)$ and $\lambda(v_B)$ computed by the FEM program applying the Ramberg–Osgood material model.

The main purpose of the above discussed example was to demonstrate the usefulness of the identified NMM-RO model for the analysis of other boundary value problems with the uniaxial stress state.



Members:
internals webs and vertical members: A=0.0625m^2,
upper chord members and border webs: A=0.3125m^2,
lower chord members: A=0.1875m^2

Fig. 5. A new truss

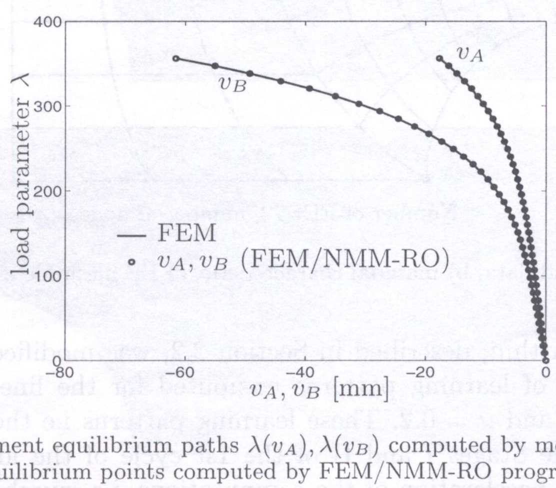


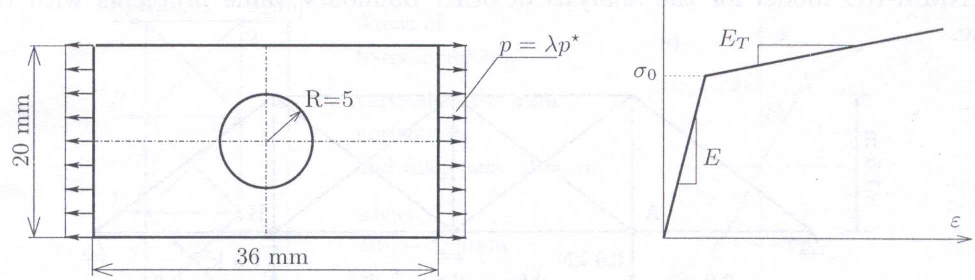
Fig. 6. Pseudo-measurement equilibrium paths $\lambda(v_A)$, $\lambda(v_B)$ computed by means of FEM program and equilibrium points computed by FEM/NMM-RO program

3.2. Elastoplastic plane stress strip

The formulation of NMM models was analysed in many papers, cf. reference in [20, 21]. In [21] an ANN (Artificial Neural Network) model was applied to the simulation of RMA (Return Mapping Algorithm). The sets of patterns for the ANN 'off line' training and testing were generated for an elastoplastic material model applying the classical theory of plasticity (Huber–Mises–Hencky yielding surface with isotropic strain hardening, associative plastic flow rule). These patterns were generated by means of special numerical procedures. The NMM model formulated in this way was very general so various boundary value plane stress problems were analysed in [21].

In the present paper the autoprogressive algorithm was applied related to the 'on line' model of pattern generation and NMM retraining. The identification process was performed for the tension perforated strip. In Fig. 7, taken from [21], the geometry, material characteristic, load data and applied FE mesh are shown. Because of two-axial symmetry a quarter of the strip was analysed. The 8-node isoparametric plane FEs with four Gauss integration points were applied. The horizontal displacement u_A at the control point A (shown in Fig. 7c) was numerically simulated using a FEM program with assumed elastoplastic material model. In such way the pseudo-measurement equilibrium path $\lambda(u_A^{\mathrm{m}})$ was formulated.

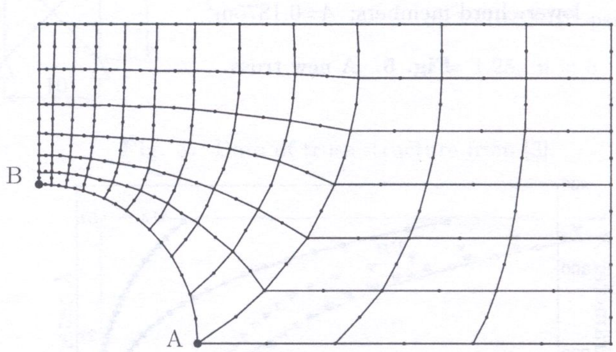
a)



$$E = 7 \cdot 10^4\ \mathrm{N/mm^2}, \quad E_T = 2.24 \cdot 10^3 \mathrm{N/mm^2}, \quad \chi = E_T/E = 0.032$$

$$\sigma_0 = 243 \mathrm{N/mm^2}, \quad \nu = 0.2, \quad p^\star = 121.5 \mathrm{N/mm^2}$$

b)



Number of FE=72, number of nodes=253

Fig. 7. a) Geometry and load data, b) material characteristic, c) FE mesh for a quarter of perforated strip

The autoprogressive algorithm, described in Section 2.2, was modified. An initial NMM model was retrained using the set of learning patterns computed for the linear elastic material model assuming $E = 7 \times 10^4$ MPa and $\nu = 0.2$. These learning patterns i.e the sets of stress ad strains, were accumulated during the Stages I and II of the 1st cycle of the identification process. This approach led to a significant acceleration of the computations, i.e. number of the loading cycles to retrain of the NMM model was decreased.

The initial optimal architecture of the NMM: 3-15-15-3 consisted of three neurons in input (3) and output $(1)_2$ layer, $H1 = H2 = 15$ neurons for each hidden layer. The optimal number of hidden layers neurons was evaluated on the base of numerical experiments.

The identification process was finished after the 2nd loading cycle since the criterion (6) in all the Gauss points of the FE system was fulfilled for the assumed error $\mathrm{er}u_{\mathrm{adm}} = 1\%$.

In Fig. 8 two simulated equilibrium paths are shown from which only the path $\lambda(u_A^{\mathrm{m}})$ was used as a pseudo-measurement curve for the NMM retraining. As can be seen in Fig. 8b, also the other equilibrium path $\lambda(u_B^{\mathrm{m}})$ estimates quite well the retraining process.

In Fig. 9a the distribution of the reduced stresses $\sigma_e(x, y)$ for the load parameter $\lambda = 2.0$ is shown. These stresses are computed by means of the FEM program assuming a theoretical elastoplastic material model which was used to simulate the pseudo-measurement curve $\lambda(u_A^{\mathrm{m}})$. The reduced stress distribution computed by the hybrid FEM/NMM program is shown in Fig. 9b. As shown in these figures, the distribution of stresses are similar for both material models.

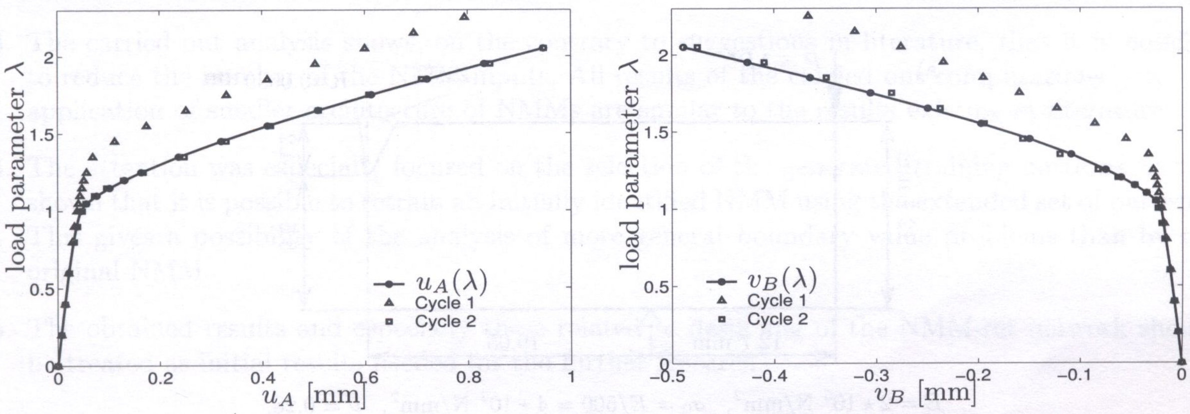


Fig. 8. Equilibrium paths $\lambda(u_A^{\mathrm{m}})$, $\lambda(u_B^{\mathrm{m}})$ and equilibrium points evolution during training of NMM process

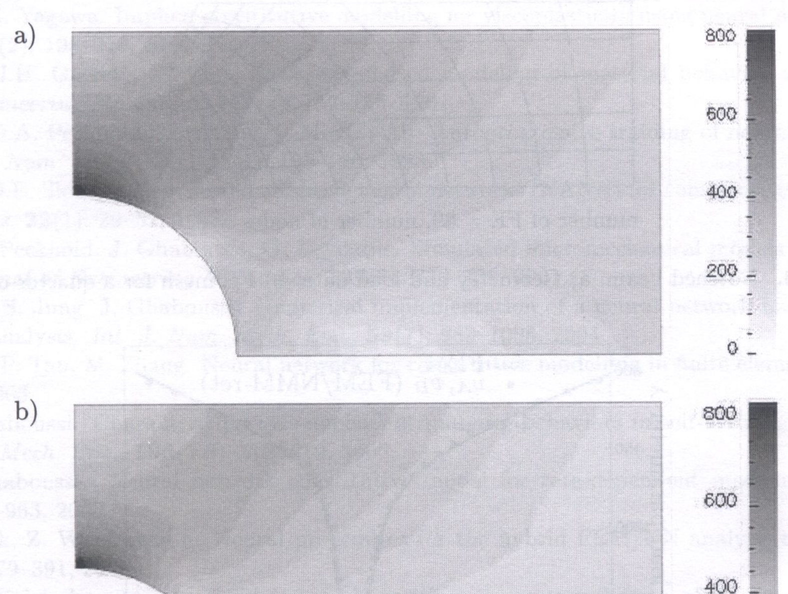


Fig. 9. Distribution of reduced stress $\sigma_e(x, y)$ in the strip shown in Fig. 8b for load parameter $\lambda = 2.0$, computed for: a) elastoplastic material model, b) identified NMM model

In paper [21] a network NMM: 8-40-20-3 was trained 'off line' applying the set of patterns generated for a theoretical elastoplastic material model, mentioned above. The NMM model was used not only for the analysis of the tension perforated strip but also to the analysis of bending of a notched beam, cf. Fig. 10.

In order to perform the analysis of the notched beam by the hybrid FEM/NMM program, the same network NMM: 3:15:15:3, which was identified for the tension strip (Fig. 7), was retrained. The retraining process was carried out using an extended set of patterns. This set consists of patterns generated during tension and compression process of the strip in Fig. 7, using FEM/NMM hybrid program. The retrained model NMM-ret turns out to be more general than the original NMM without retraining.

The model NMM-ret was first of all verified on a tension strip example. The results of a such verification were very satisfactory since the results obtained by the FEM/NMM-ret program are the same as the results shown in Fig. 8. Now this hybrid program was applied to the analysis of a new boundary value problem related to the notched beam.

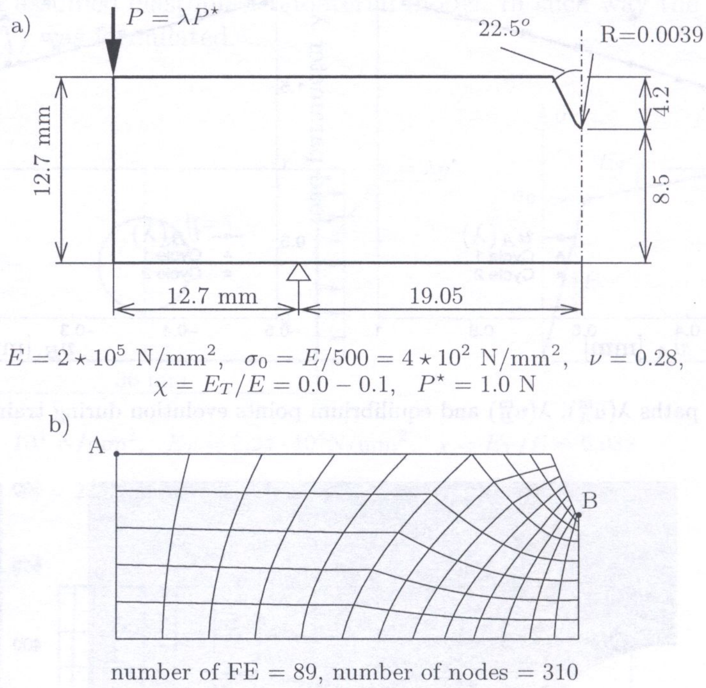


Fig. 10. Notched beam: a) Geometry and load data, b) FE mesh for a quarter of beam

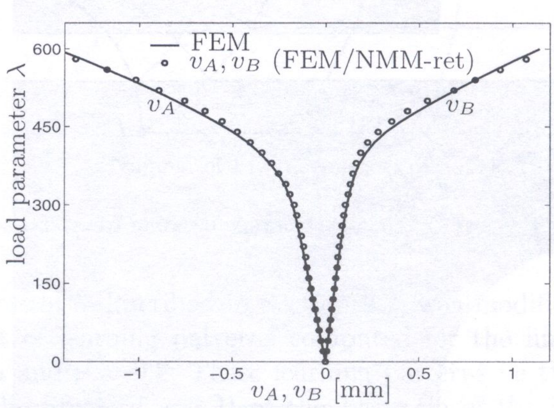


Fig. 11. Pseudo-measurement equilibrium paths $\lambda(v_A)$ and $\lambda(v_B)$ computed by FEM program and equilibrium points computed by hybrid FEM/NMM-ret program

In Fig. 11 the equilibrium points of the vertical displacements v_A and v_B are drawn. As shown in this figure the equilibrium points obtained from FE/NMM-ret program are placed very close to the equilibrium paths $\lambda(u_A)$ and $\lambda(u_B)$, calculated by means of the FEM program with elastoplastic material model using data shown in Fig. 7.

4. Final remarks and conclusions

1. Application of neural networks to the identification of equivalent materials in real structures and continua opens the door to formulation of new nondestructive methods basing on measurements of displacements on structural level.
2. The autoprogressive algorithm was efficiently applied to the design of neural based material models (NMMs). NMMs can be applied to simulate the stress-strain relations and they enable calculation of the material constitutive matrix.
3. The carried out analysis shows, on the contrary to suggestions in literature, that it is possible to reduce the number of the NMM inputs. All results of the carried out computations with the application of smaller architecture of NMMs are similar to the results existing in literature.
4. The attention was especially focused on the selection of the generated training patterns. It was shown that it is possible to retrain an initially identified NMM using the extended set of patterns. This gives a possibility of the analysis of more general boundary value problems than by the original NMM.
5. The obtained results and especially these related to designing of the NMM-ret network should be treated as initial results needed for the further research.

References

[1] T. Furukawa, G. Yagawa. Implicit constitutive modelling for viscoplasticity using neural networks. *Int. J. Num. Meth. Eng.*, **43**(2): 195–219, 1998.

[2] J. Ghaboussi, J.H. Garrett, X. Wu. Knowledge-based modeling of material behavior with neural networks. *Journal of Engineering Mechanics*, **117**(1): 132–153, 1991.

[3] J. Ghaboussi, D.A. Pecknold, M. Zhang, R.M. Haj-Ali. Autoprogressive training of neural network constitutive models. *Int. J. Num. Meth. Eng.*, **42**(1): 105–126, 1998.

[4] J. Ghaboussi, D.E. Sidarta. New nested adaptive neural networks (NANN) for constitutive modelling. *Computer and Geotechnics*, **22**(1): 29–52, 1998.

[5] R. Haj-Ali, D. Pecknold, J. Ghaboussi, G. Voyiadjis. Simulated micromechanical models using artificial neural networks. *Journal of Engineering Mechanics*, **127**(7): 730–738, 2001.

[6] Y.M. Hashash, S. Jung, J. Ghaboussi. Numerical implementation of a neural network based material model in finite element analysis. *Int. J. Num. Meth. Eng.*, **59**(7): 989–1005, 2004.

[7] A.A. Javadi, T.P. Tan, M. Zhang. Neural network for constitutive modelling in finite element analysis. *CAMES*, **10**: 523–529, 2003.

[8] S. Jung, J. Ghaboussi. Characterizing rate-dependent material behaviors in self-learning simulation. *Comput. Methods Appl. Mech. Eng.*, **196**(1-3): 608–619, 2006.

[9] S. Jung, J. Ghaboussi. Neural network constitutive model for rate-dependent materials. *Comput. Struct.*, **84**(15-16): 955–963, 2006.

[10] Ł. Kaczmarczyk, Z. Waszczyszyn. Neural procedures for the hybrid FEM/NN analysis of elastoplastic plates. *CAMES*, **12**: 379–391, 2005.

[11] M. Lefik, M. Wojciechowski. Artificial Neural Network as a numerical form of effective constitutive law for composites with parametrized and hierarchical microstructure. *CAMES*, **12**: 183–194, 2005.

[12] M Lefik, B.A. Schrefler. Artificial neural network for parameter identifications for an elasto-plastic model of superconducting cable under cyclic loading. *Comput. Struct.*, **80**: 1699–1713, 2002.

[13] M Lefik, B.A. Schrefler. Artificial neural network as a incremental non-linear constitutive model for a finite element code. *Comput. Methods Appl. Mech. Eng.*, **192**: 3265–3283, 2003.

[14] E. Pabisek. On algorithms for identification of a neural network based model of equivalent material in real structures. *Archives of Civ. Eng.*, 2007. (accepted for publication).

[15] H.S. Shin, G.N. Pande. On self-learning finite element codes based on monitored response of structure. *Computer and Geotechnics*, **27**: 161–178, 2000.

[16] H.S. Shin, G.N. Pande. Identification of elastic constants for orthotropic materials from a structural test. *Computer and Geotechnics*, **30**: 571–577, 2003.

[17] D.E. Sidarta, J. Ghaboussi. Modelling constitutive behavior of materials from non-uniform material tests. *Computer and Geotechnics*, **22**(1): 53–71, 1998.

[18] J.C. Simo, R.L. Taylor. A return mapping algorithm for plane stress elastoplasticity. *Int. J. Num. Meth. Eng.*, **22**(3): 649–670, 1986.

[19] T. Strzelecki *et al. Mechanics of Nonhomogenous Continua. Homogenisation Theory* (in Polish). Wydawnictwo Naukowe, 1996.

[20] Z. Waszczyszyn. Artificial neural networks in civil and structural engineering: Ten years of research in Poland. *CAMES*, **13**(4): 489–512, 2006.

[21] Z. Waszczyszyn, E. Pabisek. Hybrid NN/FEM analysis of the elastoplastic plane stress problem. *CAMES*, **6**: 177–188, 1999.

[22] Z. Waszczyszyn, E. Pabisek. Neural network supported FEM analysis of elastoplastic plate bending. In *BUTE Research News*, pages 12–19, Budapest, Hungary, 2000.

[23] Z. Waszczyszyn, L. Ziemiański. Neurocomputing in the analysis of selected inverse problems of mechanics of structures and materials. *CAMES*, **13**: 125–159, 2006.

[24] G. Yagawa, H. Okuda. Neural networks in computational mechanics. *Archives of Comp. Methods in Eng.*, **4**: 435–512, 1996.