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Application of swarm intelligence algorithms in solving the inverse heat conduction problem

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In the paper a proposal of using selected swarm intelligence algorithms for solving the inverse heat conduction problem is presented. The analyzed problem consists in reconstructing temperature distribution in the given domain and the form of heat transfer coefficient appearing in the boundary condition of the third kind. The investigated approaches are based on the Artificial Bee Colony algorithm and the Ant Colony Optimization algorithm, the efficiency of which are examined and compared.

Keywords: Swarm Intelligence, ACO algorithm, ABC algorithm, Inverse Heat Conduction Problem.

1. Introduction

Algorithms of Swarm Intelligence, including the Artificial Bee Colony algorithm (ABC) and the Ant Colony Optimization algorithm (ACO), belong to the group of optimization algorithms inspired by the behavior of natural systems existing and functioning in real world. Indisputable advantages of algorithms of this kind are their effectiveness, relative simplicity and universality, as well as the fact that the only assumption needed by those algorithms is the existence of solution. ABC and ACO algorithms are inspired by the techniques of searching for food by the colony of insects – bees or ants, respectively. Collective behavior of these simple non-intelligent creatures is organized in such a way that action of each individual, not aware of the common goal, is a small part of procedure leading to the success of entire swarm. In this paper we present two procedures based on the ABC and ACO algorithms used for solving the inverse heat conduction problem, i.e. a heat conduction problem with an incomplete set of input data consisting in determination of the function describing the distribution of temperature and in reconstruction of some of the boundary conditions [1, 14].

Solution of the inverse problem is much more difficult than solution of the direct heat conduction problem in which the initial and boundary conditions are known, only the temperature must be found. However, some methods for solving the inverse problem are proposed, for example the boundary elements method [11, 13], mollification method [12], Monte Carlo method [6], methods applying the Green's functions [2], wavelets theory [18] or genetic algorithms [16]. In papers [7, 8] the authors have used the ABC and ACO algorithms, respectively, for minimizing the functional representing the crucial part of approach leading to the solution of the inverse heat conduction problem consisting in heat flux reconstruction.

In the current paper the inverse heat conduction problem with boundary condition of the third kind is analyzed. This means that the distribution of temperature needs to be determined and the form of heat transfer coefficient appearing in boundary condition of the third kind reconstructed. The problem of the heat transfer coefficient identification was already investigated, for example in [5, 15, 17], however, the idea of using the ACO and ABC algorithms for solving this problem is new.

2. Abc algorithm

The Artificial Bee Colony algorithm imitates the technique of searching for nectar around the hive and communication between bees. After discovering the attractive source of food the bee (called the scout) flies back with a sample of nectar to the hive where it informs the other bees (called the viewers) with the aid of special dance, called the waggle dance. The dance of bees takes place in the special part of the hive (near the exit) and consists of two parts: moving straight and moving back to the starting point, along the semicircle, once to the right side, next to the left side. During the straight movement the bee body swings, once to the right side, once to the left side, and it emits buzzing sounds produced by very quick movements of the bee wings. Direction of the bee dance determines the angle between the localized source of food and the sun. By taking into account the fact that the position of the sun changes during the day, the bee modifies the angle of its straight movement about 1 degree every 4 minutes. Duration of the straight movement determines the distance between the hive and the source of food (each 75 miliseconds of moving straight denotes 100 meters of distance). Magnitude of the bee body vibration during the dance indicates the quality of nectar.

The ABC algorithm is initialized by random selection of vectors $\mathbf{x}_i$ from among all the vectors available in the investigated domain. The selected vectors imitate localization of the sources of nectar. The smaller the value of minimized function for the given vector $\mathbf{x}_i$, the better the quality of the source designated by this vector. In this way the best quality vector is chosen. In the first part of the algorithm positions $\mathbf{x}_i$ are modified, an assumed number of times, according to the relevant formula. This should imitate some control movements made by the bee-scout around the chosen source to check whether some better source in the neighborhood can be found. After each modification the value of minimized function is calculated to verify whether some smaller value than in the previous step is obtained. The second part of the algorithm imitates the actions of bee-viewers. Some of the vectors, fixed in the first part of the algorithm, are chosen with relevant probabilities, the bigger the better is the quality of the given vector. The selected localizations are again modified according to the same procedure as in the first part of the algorithm. Next, the best localization for the entire cycle is selected. The described cycle of the algorithm is repeated an assumed number of times.

A more detailed description of ABC algorithm is presented in papers [7, 9, 10].

3. Aco algorithm

The Ant Colony Optimization algorithm was inspired by the observation of the behavior of the real ant community and its technique of finding the shortest way connecting the ant-hill with the source of food by passing round the obstacles appearing on the ants' way. These almost blind creatures can follow one another thanks to the pheromone, choosing in this way the best of the possible trails. Pheromone is a chemical substance produced and recognized by most of ant species. This substance is left in the ground by the moving ant and afterwards it is smelled by the other ants, owing to which the other ants can follow the trace made by the first one. The stronger the pheromone trace, the greater number of ants will choose the trail covered by it. According to this, the shorter the trail to the source of food, the faster this trail will be traversed by the ant. Next, the ant returns to the ant-hill using the same trail and intensifying the pheromone trace. The pheromone trace placed on a longer trail will gradually evaporate.

Initialization of the ACO algorithm is similar to the ABC algorithm. Vectors $\mathbf{x}_i$, randomly dispersed in the investigated region, play the role of artificial ants. One of the ants, for which the minimized function takes the lowest value, is selected as the best one. The main part of the algorithm consists in improving localizations of the ants. The vector of each ant is updated at the beginning of each iteration by adding some vector **dx** determining the length of jump to the best ant localization. Elements of **dx** are randomly generated from interval $[-\beta, \beta]$ (where $\beta = \beta_0$ is the narrowing parameter defined in the algorithm). At the end of each iteration the narrowing parameter β is reduced by multiplication by 0.1. This procedure simulates evaporation of the pheromone trail in nature. In this way, the ants-vectors condense around the point of the lowest function value. The described procedure is iterated until the assumed maximal number of iterations has been achieved.

A detailed scheme of the Ant Colony Optimization algorithm can be found in [3, 4, 8].

4. Formulation of the problem

Let us consider the heat conduction equation with the following boundary and initial conditions

$$c\,\rho \frac{\partial u}{\partial t}(x,t) = \lambda \frac{\partial^2 u}{\partial x^2}(x,t), \qquad x \in [0,d], \quad t \in [0,T], \tag{1}$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \qquad t \in [0,T], \tag{2}$$

$$u(x,0) = u_0, \qquad x \in [0,d], \tag{3}$$

where c is specific heat, ρ denotes mass density, λ is thermal conductivity and u, t and x refer to temperature, time and spatial location. The investigated problem consists in identification of the heat transfer coefficient α appearing in the following boundary condition of the third kind defined on boundary for $x = d$:

$$-\lambda \frac{\partial u}{\partial x}(d,t) = \alpha\big(u(d,t) - u_\infty\big), \qquad t \in [0,T], \tag{4}$$

where u_∞ describes the temperature of environment. Another unknown element is temperature distribution $u(x,t)$ in the analyzed region. For the given value α of heat transfer coefficient the problem, defined by equations (1)–(4), turns into a direct problem, whose solution enables finding the values of temperature $u(x,t)$.

In the inverse problem in question we know the values of temperature $u(x_i, t_j)$, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, m$, in selected points of the domain and we desire to determine the value of heat transfer coefficient α. The proposed approach consists in solving the direct heat conduction problem, described by equations (1)–(4), by taking the value of heat transfer coefficient as an unknown parameter α. Thus, the received solution $\widetilde{u}(x_i, t_j)$ depends on parameter α. Next, we determine the value of α by minimizing the following functional

$$P(\alpha) = \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{m} \big(u(x_i,t_j) - \widetilde{u}(x_i,t_j)\big)^2}, \tag{5}$$

representing the differences between obtained results $\widetilde{u}$ and given values u of temperature in measurement points. To minimize functional (5) we use the ABC and ACO algorithms, respectively.

5. Experimental results

Theoretical considerations will be illustrated by an example, in which $c = 1000\,[\mathrm{J/(kg \cdot K)}]$, $\rho = 2679\,[\mathrm{kg/m^3}]$, $\lambda = 240\ [\mathrm{W/(m \cdot K)}]$, $T = 1000\,\mathrm{s}$, $d = 1\,\mathrm{m}$, $u_0 = 1013\,\mathrm{K}$ and $u_\infty = 298\,\mathrm{K}$. The exact value of the sought parameter is $\alpha = 28\,[\mathrm{W/(m^2 \cdot K)}]$. To construct functional (5) we use the exact values of temperature, determined for the known exact α, and values noised by the random error of 1, 2 and 5%.

In Table 1 the relative errors of sought heat transfer coefficient identification obtained for undisturbed input data are compiled. The calculations were made for six ants and six bees in the respective swarm intelligence algorithm and for successive numbers of iterations in each algorithm. The presented errors indicate that the ACO algorithm is convergent to the exact solution faster than the ABC algorithm, however, the approximation received with the aid of the ABC algorithm is also satisfying starting from, for example, the 7th iteration.

Table 1. Relative error of parameter α reconstruction for undisturbed input data, for six individuals in each algorithm and various numbers of iterations.

Iterations	$\text{Error}_{\text{ACO}}$ [%]	$\text{Error}_{\text{ABC}}$ [%]
2	4.6196	1563.40004
3	0.0217	74.6747
4	0.0147	0.6079
5	0.0011	0.0636
6	$4.30 \cdot 10^{-5}$	0.2195
7	$2.7502 \cdot 10^{-6}$	0.0467
8	$2.4823 \cdot 10^{-7}$	0.0245
9	$3.2859 \cdot 10^{-8}$	0.0020
10	$7.8858 \cdot 10^{-10}$	0.0001

Above conclusion is confirmed by the results collected in Table 2 which presents the relative errors of parameter α reconstruction calculated for various sizes of noise of the input data, for six individuals in each algorithm, but for the number of iterations smaller by half in case of the ACO algorithm (5 iterations) in comparison with the ABC algorithm (10 iterations). As can be seen, approximation errors are comparable in both cases, smaller than input data error for 1% and 2% noise and at the level of input data error in case of 5% noise.

Table 2. Relative error of parameter α reconstruction for disturbed input data, for six individuals in each algorithm, for five iterations in ACO and ten iterations in ABC.

Input data error	$\text{Error}_{\text{ACO}}$ [%]	$\text{Error}_{\text{ABC}}$ [%]
1%	0.3414	0.3965
2%	0.6414	0.6532
5%	5.0102	5.1021

The calculations indicate that optimal parameters for the compared algorithms are: six ants and five iterations for the Ant Colony Optimization algorithm, five bees and ten iterations for the Artificial Bee Colony algorithm. Figures 1 and 2 show the comparisons of the exact values of temperature on boundary $x = 1$, where the third kind condition is reconstructed, with its approximate values (for input data perturbed by the error of size 2%) calculated by applying the ACO and ABC algorithms for their optimal parameters, respectively. Distribution of the relative errors of these reconstructions are also given. A similar comparisons but for 5% error of input data are compiled in Figs. 3 and 4. The presented results show that both algorithms for their optimal parameters assure very good approximations of exact distribution of temperature. Maximal errors of approximations are much smaller than the errors of input data. The time needed for executing one iteration is similar for both investigated algorithms, however, the ACO algorithm needs smaller by

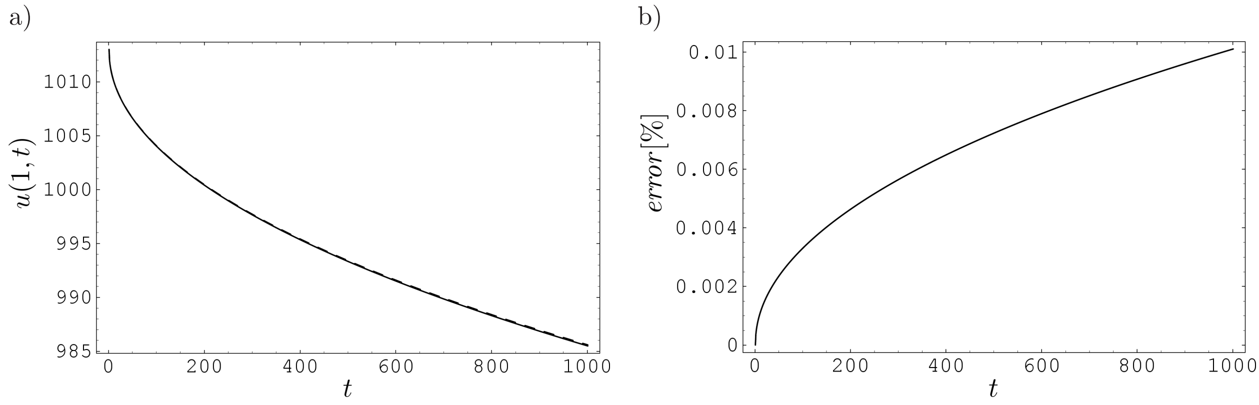


Fig. 1. Distribution of temperature $u(x,t)$ for $x = 1$ obtained using ACO algorithm for 2% error of input data (a): solid line – exact solution, dashed line – approximated values) and error of this approximation (b).

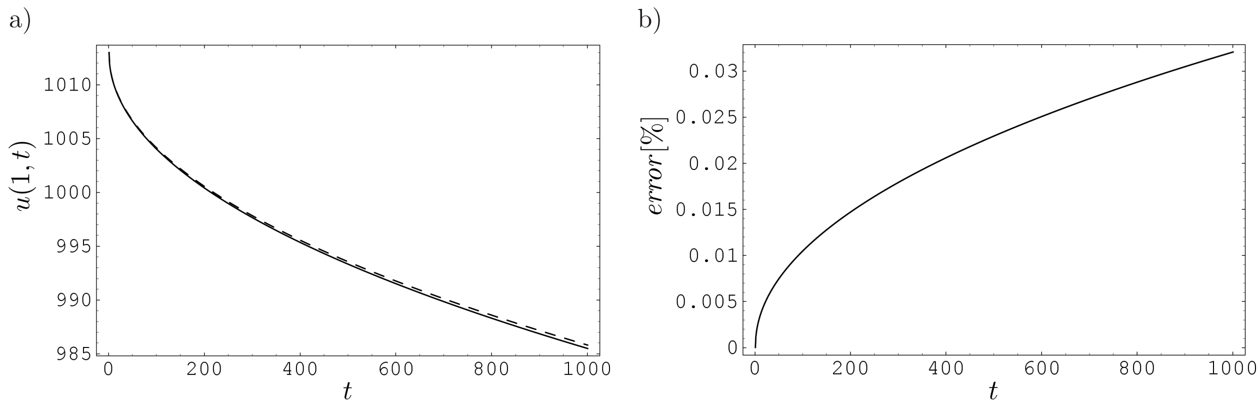


Fig. 2. Distribution of temperature $u(x,t)$ for $x = 1$ obtained using ABC algorithm for 2% error of input data (a): solid line – exact solution, dashed line – approximated values) and error of this approximation (b).

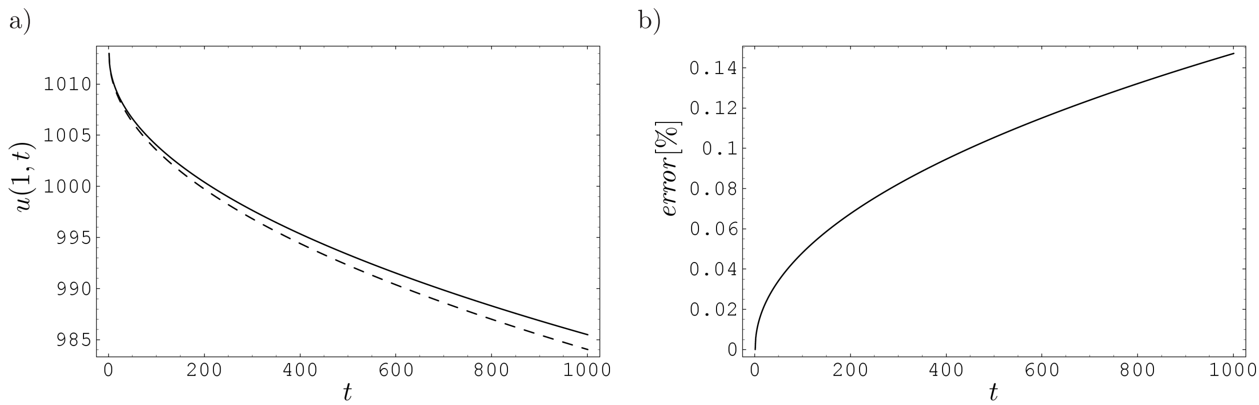
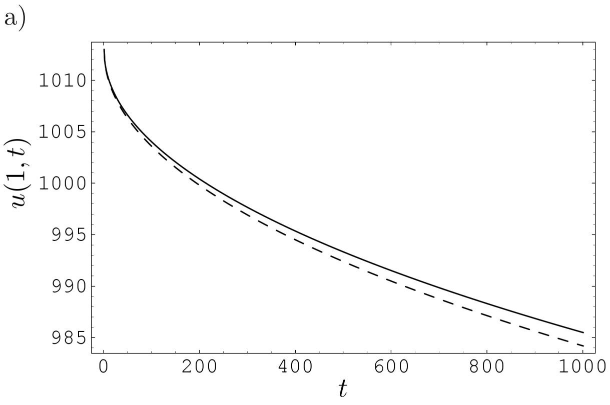


Fig. 3. Distribution of temperature $u(x,t)$ for $x = 1$ obtained using ACO algorithm for 5% error of input data (a): solid line – exact solution, dashed line – approximated values) and error of this approximation (b).

half the number of iterations than the ABC algorithm to obtain satisfying approximation. Therefore, in view of the fact that each iteration of the algorithm means the necessity of solving the appropriate direct heat conduction problem, we point at the Ant Colony Optimization algorithm as slightly more useful for solving the considered inverse heat conduction problem.



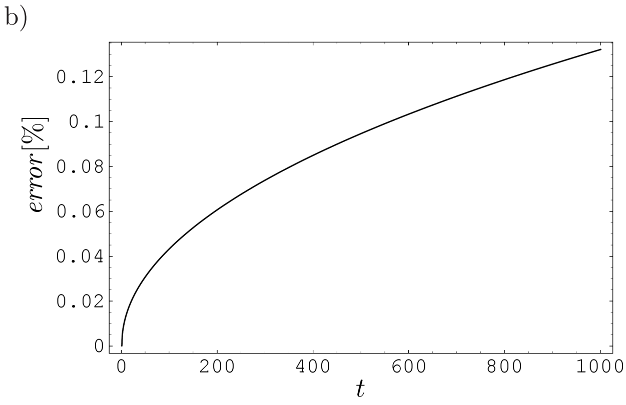


Fig. 4. Distribution of temperature $u(x, t)$ for $x = 1$ obtained using ABC algorithm for 5% error of input data (a): solid line – exact solution, dashed line – approximated values) and error of this approximation (b).

6. Conclusions

In the paper we have presented the comparison of two procedures used for solving the inverse heat conduction problem with boundary condition of the third kind. The solution of this problem consisted in identification of the heat transfer coefficient and reconstruction of the temperature distribution in the analyzed region. An important part of the procedure was minimization of the functional expressing the errors of approximate results. To minimize this functional we used in the first procedure the Artificial Bee Colony algorithm and in the second procedure the Ant Colony Optimization algorithm. The two approaches were compared with regard to the precision of received approximate solution as well as to the needed number of calculations of the objective functional value.

Summarizing, we may conclude that both swarm intelligence algorithms are useful for solving the considered problem, they give satisfying results for small numbers of individuals as well as for relatively small numbers of iterations. However, taking into account the number of calculations indispensable to obtain good results, which indicates the velocity of working of the algorithms, the ant algorithm appeared to be slightly more efficient in solving this kind of problem. The number of iterations in the ACO algorithm execution, implying the number of direct heat conduction problems to be solved, is smaller by half in comparison with the ABC algorithm.

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