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Artificial Neural Network as a numerical form of effective constitutive law for composites with parametrized and hierarchical microstructure

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In the paper, Artificial Neural Network with hidden layers is used to approximate the functional dependence of the effective properties of a composite on the physical properties of its micro-components. Two numerical examples have been examined in order to demostrate this approach. The first one introduces geometrical parameters of the cell of periodicity into ANN training process. It proves the ability of ANN to catch the behaviour of the composite material based on the properties of the components and their spatial arrangement at micro level. The second example deals with a special case of the self-repetitive composite structure. It has been shown that, in the limit, the geometry and behaviour of such a composite is consistent with the fractal form known in the literature as the Sierpinski's carpet.

Keywords: neural network, homogenisation, hierarchical composite

1. Introduction

In the previous works [2, 6, 10] of the authors an Artificial Neural Network (ANN) has been used to approximate an effective constitutive law for a composite. It was possible to define an observable at macro level material behaviour, having given constitutive description of micro-components and a micro-geometry of the composition. As it is known, the effective behaviour for composite is usually very difficult to handle by a closed form formula, except for self-consistent homogenisation. Unfortunately, this approach is not always applicable. In particular, it is not applicable in the case when the micro-geometry of the composition is complex, has a structural character and cannot be sufficiently characterised by the only volume fraction of components. Examples of such a case can be found in [6, 7]. Thus, an ANN trained with a set of examples, replaces well the symbolic definition of the constitutive relationships. In the present paper the same idea is exploited but now in the case of hierarchical composites and in a very different organisation of the approximation. In the quoted above former papers, ANN substituted the functional relationships between an increment of average stress tensor and increments of average strain tensor.

$$\Delta\boldsymbol{\sigma}^{\text{ave}} = \text{ANN@}\left\{ \boldsymbol{\sigma}^{\text{ave}} \quad \Delta\boldsymbol{\varepsilon}^{\text{ave}} \right\}. \tag{1}$$

The symbol @ in (1) denotes an action of an "ANN operator" on the ordered set of values ($\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$) are stress and strains respectively, Δ denotes an increment.

Here, the ANN approximates the functional dependence of elements of the effective constitutive tensor D_{ijkl} on the constitutive tensors $\mathbf{D}$ of each of n components of the composite and on some

scalar parameters c characterising the geometry of the microstructure:

$$D^{\text{eff}}_{ijkl} = \text{ANN@}\{\ \mathbf{D}^{(1)}\ \ ..\ \ \mathbf{D}^{(n)}\ \ c^k\ \}. \tag{2}$$

Many natural and man-made materials exhibit an internal structure at more than one length scale. The simplest case occurs when between the macro and micro levels an intermediary structure forms a meso-structural level. Materials with internal structure may show also a hierarchy of more than one, intermediary structural levels. A typical reticular structure with a huge number of bars can be seen as a three level composition: the macro behaviour can be modelled as a shell, the meso level is that of the elementary, repetitive reticular cell, the micro is the bar itself of the complex cross section or being assembled from different materials. Some abstract, fractal-like structures can be considered as composites for which the number of structural levels tends to infinity.

In this paper the ANN will be used to approximate, memorise and even discover the law governed an effective (observable at macro scale) behaviour for composites, the microstructure of which depends on some parameters of mechanical or geometrical nature. The simple consequence of this is the possibility of computation of the effective characteristics for hierarchical composites. Of course, the ANN will be used here as a suitable and powerful tool of approximation of the given knowledge on the macro behaviour of the composite. The source of knowledge must be found elsewhere. In this paper, the homogenisation theory will be used as an unique source of knowledge on the constitutive relationships between mean (observable at the macro level) mechanical and kinematical variables. The alternative would be a real or numerical experiment (see [2, 6, 10].

In what follows in this introductory part, the theory of homogenisation and the use of ANN in mechanics will be shortly summarised, finally the specific application of the ANN will be presented, which is the subject of the present paper. The introduction will be finished with some ideas of possible application of the presented numerical tool of analysis. In the following sections details of the method and some illustrative examples will be presented.

1.1. Theory of homogenisation as a source of data for ANN training

The asymptotic theory of homogenisation permits to deduce the matrix of effective material characteristics for composite from given properties of components and their spatial arrangement inside the representative volume of the heterogeneous materials. The classical approach assumes (see [?]) that the representative volume has the form of the exactly repetitive portion of the material, called "cell of periodicity". The separation of scales is assumed between the single cell and the whole of the body. The small parameter ε characterising this scale separation is computed as a ratio between characteristic lengths of macro scale and that of the cell of periodicity. This assumption, applied to the multi-scale material, requires the scale separation at each level of the structure.

Let $\mathbf{u}^0$, $\boldsymbol{\sigma}^0$ be the solution of the "homogenised" problem i.e. the problem in which the variable material coefficients $\mathbf{D}(\mathbf{y})$ are replaced with some unknown constant values $\mathbf{D}^h$. We suppose, that the periodicity of material characteristics imposes an analogous periodical perturbation on the quantities describing the mechanical behaviour of the body. Hence, for displacements we have:

$$\mathbf{u}^\varepsilon(\mathbf{x}) \equiv \mathbf{u}^0(\mathbf{x}) + \varepsilon\,\mathbf{u}^1(\mathbf{x},\mathbf{y}), \tag{3}$$

$$\text{where}\quad u^1_i(\mathbf{x},\mathbf{y}) = \chi^{pq}_i(\mathbf{y})\,\frac{\partial u^0_p(\mathbf{x})}{\partial x_q} + C_i(\mathbf{x}) \quad \text{and } \boldsymbol{\chi} \text{ is periodic on the cell.} \tag{4}$$

Two coordinate systems are used in the above: $\mathbf{x}$ for macro level, $\mathbf{y}$ for micro level (in general – for an immediate lower level in the hierarchy of composition).

The applied version of the homogenisation technique needs a solution of a boundary value problem (BVP) with periodic boundary condition posed over the cell of periodicity for each geometry of the cell and each permutation of values of the inside material properties:

find $\chi_i^{pq} \in V_Y$ such that: $\forall\, v_i \in V$

$$\int_Y D_{ijkl}(\mathbf{y}) \left(\delta_{ip}\delta_{jq} + \chi^{pq}_{i,j(y)}(\mathbf{y})\right) v_{k,l(y)}(\mathbf{y})\, \mathrm{d}\Omega = 0. \tag{5}$$

Once the so-called homogenisation function χ is found, the effective characteristics are computed according to the formula:

$$D^h_{ijpq} = |Y|^{-1} \int_Y a_{ijkl}(\mathbf{y}) \left(\delta_{kp}\delta_{lq} + \chi^{pq}_{k,l(y)}\right) \mathrm{d}Y. \tag{6}$$

Let us consider the following "from top to bottom" description of the considered hierarchical composite. At the macro level the material filling a domain Ω of unitary diameter can be considered as a homogeneous one (but is not). The whole domain Ω is made of a huge number of periodically repeated, heterogeneous cells Ω^1 of the diameter ε, small with respect to unity. Scaling the cell by $(1/\varepsilon)$ reveals that it is composed of two materials of the first level of composition (n materials in general, we limit ourselves to $n = 2$). Each of them can be considered as homogeneous.

Any i-th of the two materials filling the cell Ω^1 of the first sub-structural level can be either homogeneous or made of a huge number of periodically repeated heterogeneous cells Ω^2_i of the diameter of order ε^2. Two types of cells of periodicity define a second sub-structural level. Each of the cells can be treated then in the same manner as it was before with the cell Ω^1 in order to discover a deeper structural level. For each of the levels the analysis analogous to the one described by equations (3)–(6) can be formulated. In the case of the lower level, next to that the equation (3) concerns. This analysis starts with decomposition:

$$w^\varepsilon(y) \equiv w^0(y) + \varepsilon\, w^1(y,z) \quad \text{where} \quad w^0(y) = u^1(y, x_{\text{fixed at higher level}}) \tag{7}$$

Boundary value problems must be formulated now for the cell of periodicity of the lower level and for the corresponding homogenization function, the lower order, local coordinate system being $\mathbf{z}$.

It is assumed that a micro level exists. At the micro level all the components of all the cells of periodicity are truly homogeneous. The homogenization procedure starts at the micro level and computes the effective material properties for the immediately higher level.

All the effective characteristics computed in the examples follow the algorithm above outlined and are interpreted in the formalism of the Finite Element Method.

The "production" of examples for the ANN training is thus time consuming, but any other use of this technique inside the computational algorithm is time consuming as well. ANN used as a "functional formula" replaces the solution of the BVP thus allows to spare the time of computations in the case of multilevel composites.

1.2. Application of ANN as an approximation tool in mechanics

The idea of using a neural network in constitutive modelling was originally proposed by Ghaboussi in [3]. The experience of the authors in representation of effective constitutive law for composite bodies is reported in [9, 10]. In these papers the ANN model is incorporated into a FE code as a kind of material description. In all these papers the most popular ANN of Back Propagation type is widely used. The paper [13] gives the state of art surveys.

From a physical point of view, ANN can be considered as a collection of some simple processing units (nodes) that are mutually interconnected by connectors with adjustable weights. This system of logical or physical units (i.e. elements of a computer software or hardware) is organised to transform an input signal into an output signal. For an exhaustive and fundamental introduction to the ANN technique the reader is referred to any of the textbooks: [4, 5, 11]. The ANN is trained by means of the BP (Back Propagation) algorithm. According to this method, the weights of connections

are enhanced iteratively by successive corrections, proportional to the error, which is transmitted through the link. This process is called "training" and is continued until the error between the neural network output and the desired (known) output is minimised for a whole set of pairs: given input – known output.

A non-symbolic, neural model is constructed as follows: the ANN is trained first to reflect correctly a set of data acquired from an experimental test. Then the network capability for automatic generalisation (interpolation between some data sets) allows us to interpret the ANN output for the input data not presented to the network.

In the application which will be presented, the use of ANN is justified by a set of theorems (by various authors, see for example [1]) which asserts, that ANN is an universal approximator of a function of many variables, a functional or an operator. Because of this, the functional dependencies between effective properties of the composite and the characteristics of the components of the cell of periodicity can be certainly handled with sufficiently trained ANN.

1.3. Approximation of dependence of effective properties on the micro and meso structural data

As it is stated in (2), independent variables are here the mechanical properties of the components and some parameters invented to describe its geometrical repartition in the cell. The functions to be approximated:

$$D^{\text{eff}}_{ijkl} = D^{\text{eff}}_{ijkl}\left(\mathbf{D}^{(1)} \;\; .. \;\; \mathbf{D}^{(n)} \;\; c^k \right) \tag{8}$$

are known from examples furnished by direct application of the homogenisation procedure for the exemplary cells.

The Artificial Neural Network for the approximation is constructed as follows:

- First group of neurons of input layer are valued with the given values of constitutive parameters of the materials of the single micro-structural cell. The second group of neurons at the input are interpreted as parameters describing the geometry of the micro-structural cell. The geometry of the micro structure must be, of course, "parametrisable" by few parameters only.
- Output layer contains neurons valued with values of effective constitutive parameters of the "homogenised" material.
- Hidden layers are constructed to assure the best approximation of the unknown relation between the material properties of components, their geometrical organisation and effective material properties at the output. The best approximation is understand in the usual sense and measured by the test and training errors. It is limited by the number and quality of the data used in training.

In the elasticity the number of the input parameters varies between: 2 time the number of materials plus number of geometrical parameters for isotropic components and: 21 time the number of materials plus number of geometrical parameters for anisotropy. It is to note, that two material parameters at the input are applicable only when the input data are from the micro level. Number of the output parameters depends on the type of effective constitutive relationships. This is theoretically known a priori. The maximum number is 21 parameter but only effective orthotropy in 3D with 9 parameters was tested by the authors, up to now. It is to note, that for a case of porous material, the void treated as the second material do not requires any additional input neuron. If the geometry of all the levels is obtained by a scaling of the same figure, the geometrical parameters in the input layer can also be omitted.

The following algorithm is proposed to perform the approximation of the effective characteristics of the composite:

1. Preparation of the learning data: for casual values of the materials data and for each kind of cell of periodicity (geometry) the effective material characteristics are computed by a FE solution of BVP with periodic conditions suitably post-processed.

2. Training of the network with the pairs of sets: given random input and computed (as said above), corresponding output. Interpretations of input and output data are defined in Sec. 2.1.2.

3. Having the well trained ANN, starting at the micro-level, for each structural level:

   - for each kind of cell of periodicity at the current level:
     - run the neural network in the recall mode with input data characterising the current level of the structure,
     - complete the sets of input data for each cell of higher structural level from ANN outputs obtained at the previous level,
   - for each cell of periodicity at the next (higher) level of composition:
     - run the same neural network in the recall mode with suitably completed input data plus information characterising the geometry of the cell of periodicity of the higher structural level.

At the macro level algorithm stops.

In Fig. 1 a typical scheme of the use of ANN for two level approximation of effective characteristics is shown. The micro cell was build from two materials: bronze and a alloy of Nb and Sn. This composite made a part of a meso cell of periodicity contained also a homogeneous component that was again bronze. It is seen in Fig. 1 that nine, simple neural networks, each one specialised with one effective parameter, obtain the approximation of nine entries of effective stiffness matrix at the meso level. Because of the geometry of the micro cell, the composite part of the meso cell was expected to be orthotropic. Since the geometry of the micro level as well as the properties of the alloy was not changed, at the input of each of nine ANN_1 there are only two variable material parameter: Young and Poisson modulus of bronze that varied strongly with temperature and with

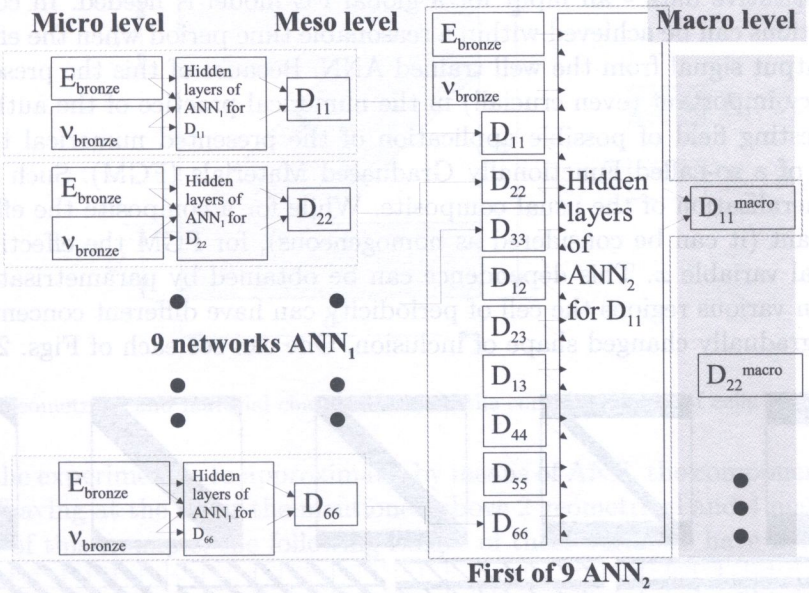

Fig. 1. Flow chart of the typical neural network application for the hierarchical composite with one intermediary (meso) level. The networks trained at the micro level (ANN_1) produce the input for the meso level: ANN_2. Thus the effective parameters for the macro scale are computed. 9 independent parameters for ortotropy are considered both for meso and macro levels

plastic yielding. At the output thus an evolution is obtained, with temperature and with yielding of each of material parameters of the heterogeneous component of the meso cell of periodicity. In the flow chart in Fig. 1 it is shown that in turn, at the meso level, nine independent ANN_2, each one with nine effective material parameters of the composite part of the cell and two (the Young and Poisson moduli) of the homogenous component, are used to approximate nine entries of the effective, global stiffness matrix (at the macro level). In this case the influence of temperature and of the material yielding is taken into account again because of variable material parameters of bronze and because of the changes of composite characteristics, discovered and approximated by ANN_1 at the micro-meso passage. The geometry of the cell is unchanged. The Figure 1 shows two level approximation of stiffness matrix that is used with very satisfying results in one of the practical applications of the authors.

1.4. Applications of the presented method

Of course, using any of homogenisation theories and applying it as many times as many levels of hierarchy there are, it is possible to compute the effective properties of a hierarchical composite without any use of ANN. In this case it is, however, necessary to solve at each step of computations (or – at each structural level) a boundary value problem for local perturbation using FE method. The procedure becomes thus time consuming. The time of a single run of computation is usually reasonable so if performed once forever for a given composite the procedure is acceptable. Unfortunately, in some recent numerical models of hierarchical composites [6, 8] the computations of effective coefficients are performed at each step of loading and in many zones of the micro-heterogeneous body. This necessity is due to the fact that locally, at the micro level, each of homogeneous components can change its mechanical properties, depending on the stress level they subject (fracturing, plastification, damage, ...). If this is a common feature for many micro-cells in a zone that can be considered as a macro domain (being still a sub-region of the considered body), a new effective properties must be calculated for this region. In practice, this is a region covered by a single element of the global FE mesh. The approach becomes impracticable if the FE solution is repeated and a suitable post-processing for each load step and for each element of the global mesh in order to obtain the effective constitutive data - an input for a global FE model is needed. In contrast, the same chain of computations can be achieved within a reasonable time period when the effective properties are read as an output signal from the well trained ANN. Because of this the presented application of the ANN is very important (even crucial!) in the numerical practice of the authors.

Another interesting field of possible application of the presented numerical techniques is theory and practice of a so-called Functionally Graduated Materials (FGM). Such materials can be considered as generalisation of the usual composite. While for a composite the effective properties are usually constant (it can be considered as homogeneous), for FGM the effective properties are functions of global variable x. This dependence can be obtained by parametrisation of the cell of periodicity thus in various regions the cell of periodicity can have different concentration of forcing in the matrix or gradually changed shape of inclusion. One can see each of Figs. 2b) as taken from

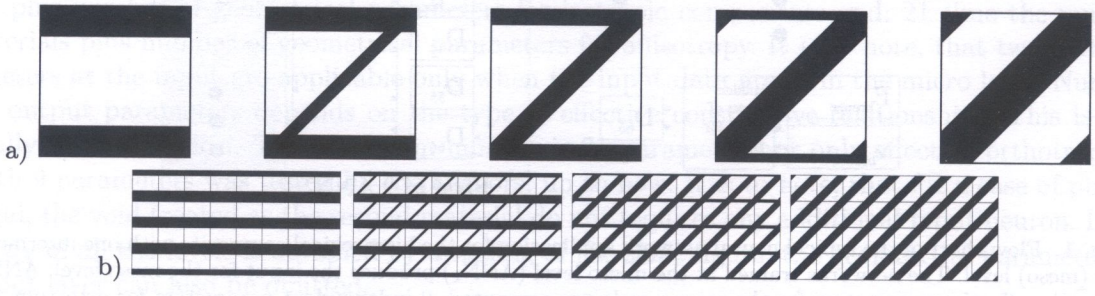

Fig. 2. a) Examples of the Z-like class of cells of periodicity, b) "smooth" transition from horizontal strata to diagonal one (Functionally Graduated Material)

different points of the FGM. Of course, if the functional dependence of geometrical parameter of the cell could be associated with functional dependence of the effective material coefficients – the optimisation and, at least, a simple FE analysis of such the structure would be numerically practicable. The use of ANN for approximation of this functional dependence in application to FGM is analysed by the authors and will be presented in a separate paper.

In the present publication the algorithm and a numerical efficiency of the proposed tool of analysis of effective properties of composites is only shown.

2. Numerical examples

2.1. ANN as a representation of a *Z*-like class of cells of periodicity

Describing the composite structure one has in mind two groups of data: material properties of the components and their spatial arrangement inside the structure. Especially, for the periodic composites, the cells of periodicity are distinguished at micro level of the structure. In case of hierarchical composites a couple of structural levels can be identified. Each level is described by a set of material and geometrical parameters. Asymptotic homogenisation allows to compute the effective parameters of the composite, but becames numerically to expensive if the component materials change their properties during loading. The geometry of the composition may be variable as well. The example shows how to deal with such kind of problems by means of ANN.

2.1.1. Preparing data for training ANN

This example deals with the class of the plain cells of periodicity shown in Fig. 2. The cells consist of two elastic materials aranged in a *Z*-like form. The internal geometry of these cells can be entirely described by two parameters: the relative thickness of the horizontal strata c_1/a and the relative thickness of the diagonal layer c_2/a. The material properties of the components are defined by elastic coefficients: E_1, ν_1 and E_2, ν_2 (see Fig. 3).

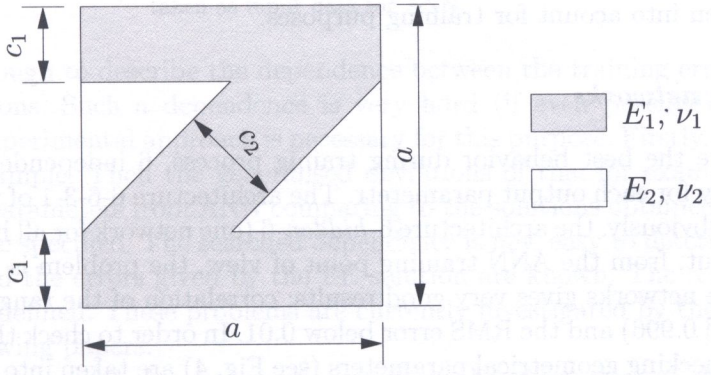

Fig. 3. Geometrical and material characteristics of the considered class of cells of periodicity

The scope of the experiment is to approximate, by means of ANN, the components of the effective elasticity matrix having at the input the mentioned above 2 geometrical and 4 material parameters. For the puropses of this example, the following ranges of these variables have been established:

$$\frac{c_1}{a} \in \left\langle 0, \frac{1}{4} \right\rangle, \qquad \frac{c_2}{a} \in \left\langle 0, \frac{\sqrt{2}}{4} \right\rangle, \qquad E_1, E_2 \in (0, 1\rangle, \qquad \nu_1, \nu_2 \in \left\langle 0, \frac{1}{2} \right). \tag{9}$$

It's expected, the designed neural network will represent the continous dependence between the variables (in their ranges) and the effective properties of the composite. In order to prepare the

training data for ANN the appropriate representations of these ranges have to be choosen. For this experiment 12 configurations of the geometrical data are taken into account as shown in Fig. 4. Obviously, the number of configurations, its distribuition and density in the space of geometrical data (2-dimensional in this case) depends on the resercher choice only. It's always a compromise between the accuracy of the ANN's approximation quality and the effort needed to preparate the data for the network and perform the training phase.

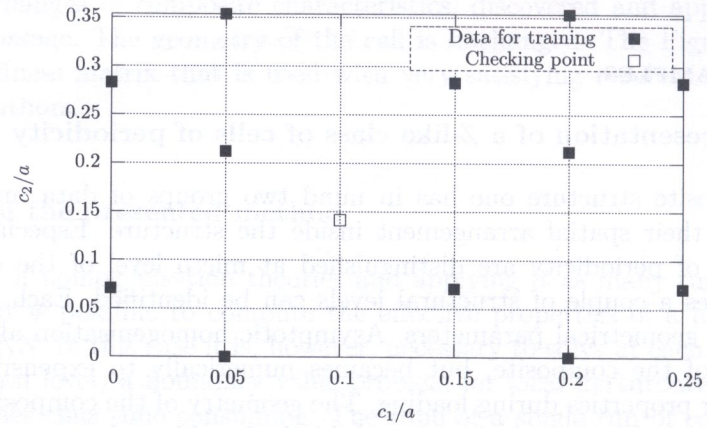

Fig. 4. Graphical representation of $(c_1/a, c_2/a)$ pairs – geometrical parameters of the cell of periodicity – used to generate the training data for ANN. The checking point will be used for testing purposes

The parameters of the component materials are generated in a random way. For each pair $(c_1/a, c_2/a)$ 50 random patterns of the material parameters E_1, ν_1, E_2, ν_2 are created. It's assumed that randomly generated parameters are distribuited in a homogeneus way in the 4-dimensional space of the material data. Finally, $12 \cdot 50 = 600$ input sets are established for training ANN. Targets for the network are obtained by means of the asymptotic homegenization method (our own code). 600 BVPs with periodic boundary conditions had to be solved in order to calculate the components of the effective $D[3 \times 3]$ matrices. 6 independent components for ortotropy: D_{11}, D_{22}, D_{33}, D_{12}, D_{13}, D_{23} are taken into acount for training purposes.

2.1.2. Training networks

In order to ensure the best behavior during trainig process, 6 independent neural networks were trained, separately for each output parametetr. The architecture 6-6-3-1 of the networks is chosen as shown in Fig. 5. Obviously, the architecture 6-*hidden*-6 (one network for all homogenized parameters) is also possible, but, from the ANN training point of view, the problem is then worse conditioned.

Training of the networks gives very good results: correlation of the range 0.995 (correlation vary between 0.994 and 0.996) and the RMS error below 0.01. In order to check the approximation quality of the ANN the checking geometrical parameters (see Fig. 4) are taken into account and the random material parameters (from the permitted ranges) were chosen. Figure 6 shows the responses of the network.

The agreement between the results given by the asymptotic homogenisation method used directly and the trained neural networks seems very good. It could be expected, the response of the ANN is of the same, good quality for all inputs from the domain defined by (9).

In addition, for testing purposes, a couple of neural networks have been trained for each component of D and the described verification data have been applied to each of them. It's observed that the correlation between the results given by the idependently trained ANN's (i.e. trained starting from the different sets of weights) is of the range $0.980 - 0.990$. It means, the starting parameters of ANN (weights) and the training process itself does not influence very much the obtained results and, finally, the FE analysis results.

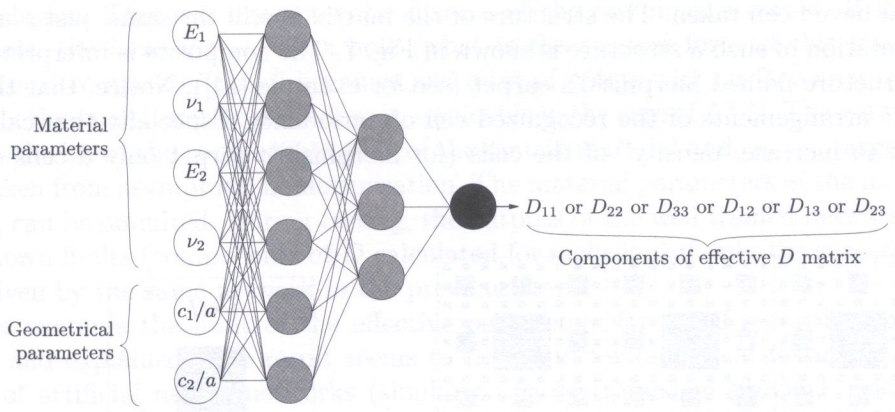

Fig. 5. Neural network of the achitecture 6-6-3-1 used for training. In the presented experiment 6 networks have been trained (separately for each component of D matrix)

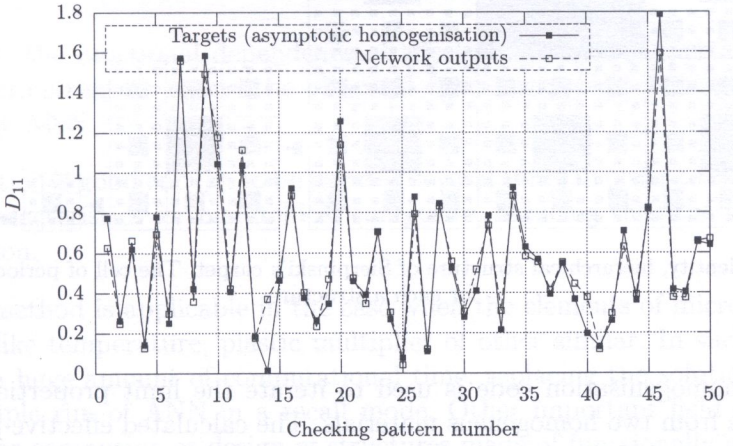

Fig. 6. Test of the network. Random material parameters and the geometry never used in training were taken as input data for ANN

Obviously, it's not enough to describe the dependence between the training errors and the finite element solution deviations. Such a dependence is very hard (if even possible) to obtain in an analytical way and the experimental àpproach is necessary for this purpose. Firstly, one should define a reliable testing FE example. Then the generalised deviations of the, for example, displacement fields calculated for the parameters from ANN comparing to the solutions obtained from asymptotic homogenisation should be specified. The searched dependence is now easy to determine as the errors given by the network and the errors given by the FE solution are known. The reliability degree of the network can also be defined. These problems are currently investigated by the authors and will be presented in the following papers.

In the described way a versatile homogenisation tool for the given class of the composite geometrical structure, i.e. for the cells of periodicity which geometry can be described by two precisely defined parameters c_1/a and c_2/a, is obtained. Such a network can be easy icorporated within a FE code instead of the homogenisation procedures. In this way the numerical costs of computations can be considerably decreased.

2.2. Hierarchical, self-repetitive composite

This example deals with a self-scaled composite material. The rectangular cell of periodicity containing a rectangular homogeneous inclusion, characterised by E and ν is taken into account. The matrix is made of a composite material except of the deepest, starting micro level, where the ho-

mogeneus material have been taken. The structure of the matrix is still the same, just scaled down. Graphical representation of such a structure is shown in Fig. 7. The composite is interpreted here as the well known structure named Sierpinski's carpet (see for example [14]). Notice, that this is only one of the possible arrangements of the recognized cell of periodicity. Especially the scaling factor might be changed to increase "density" of the cells (for Sierpinski's carpet only 8 cells constitute the next level.

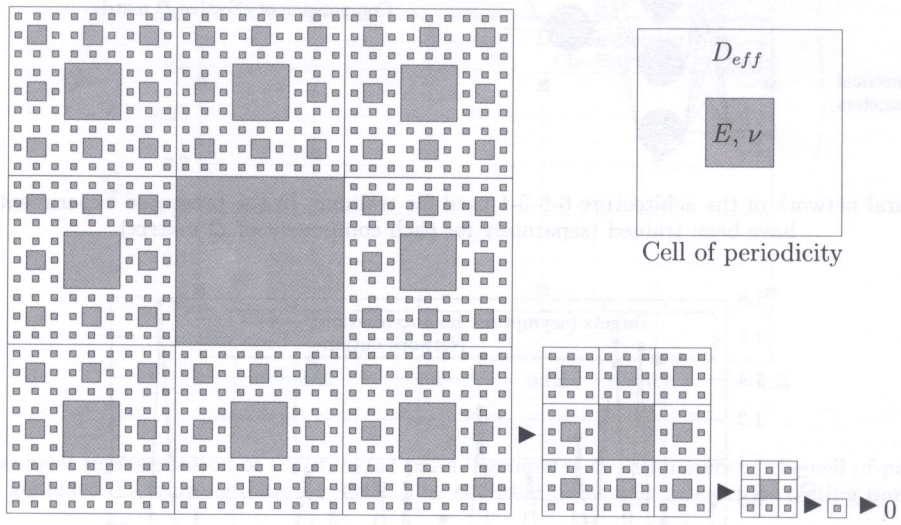

Fig. 7. The self-identity, hierarchical structure of Sierpinski's carpet. The cell of periodicity is recognized for such a structure

The asymptotic homogenisation code is used to iterate the limit properties of the composite. The procedure starts from two homogeneus materials. The calculated effective parameters are then assumed as the parameters of the matrix material for the next step. The properties of the inclusion remain unchanged. The procedure continue until the limit is achieved. As is shown in Fig. 8 the effective parameters of the composite tend to the parameters of the inclusion. This can be easy explained as it will be noticed that, in the limit, the relative area occupied by the starting homogeneus matrix material $\to 0$. In other words, the structure tends to be filled by the homogeneus material

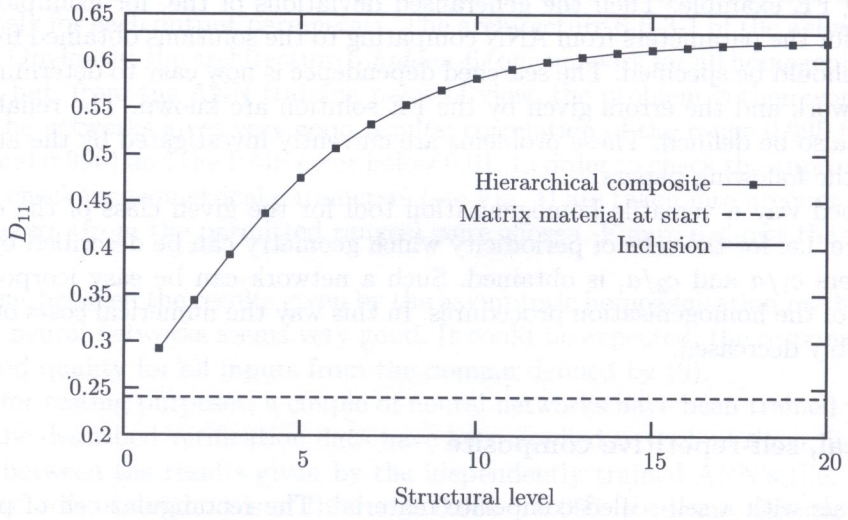

Fig. 8. Evolution of the first component of the effective D matrix. The other components of D shows the same tendency

of the inclusion. This is a characteristic feature of the Sierpinski's carpet. If it will be assumed, for example, the inclusion is just a hole (what is the original form of this fractal structure) the 2-dimensional composite "plate" becames just a set of points with surface area equal 0.

Exactly the same iterative procedure is applicable in the case of ANN. The network can be trained having as the input the components of D of the matrix material and, as the target, the components of D_{eff} taken from asymptotic homogenisation. The material parameters of the inclusion, as they are constant, can be ommited. During testing, the outputs of the well trained networks should converge to the known limits (components of D calculated for inclusion) if only the successive inputs are the results given by the same network in the previous step.

In this example the limit of the effective parameters for a self-repetitive composite has been achieved and explained. The result seems to be useful for numerical testing of correctness of the training of artificial neural networks (similar to those trained in previous example). The test is passed if the series of the computed effective values monotonically converge to known limit.

3. Conclusions

- For a composite, the functional dependence of effective material properties on parameters describing the micro-structure cannot be obtained from asymptotic homogenisation but can be approximated by ANN.
- This approximation is good enough to be iterated several times in order to compute, level by level, the effective characteristics for hierarchical composite, reducing thus the time of computations in homogenisation.
- The presented method is applicable in the case when the elements of microstructure depend on the parameter like temperature, plastic multiplier of other similar. In such a case the method allow to spare a huge amount of computational time, replacing the solution of boundary value problem by simple run of ANN in a recall mode. Other important field of application of the method would be computing or design of structures made of functionally graduated materials.

References

[1] T. Chen, H. Chen. Universal Approximation to Non-linear Operators by Neural Networks with Arbitrary Activation Functions and Its Application to Dynamical Systems, *IEEE Trans. on Neural Networks*, **6** (4): 911–917, 1995.

[2] D. Gawin, M. Lefik, B.A. Schrefler. ANN approach to sorption hysteresis within a coupled hygro-thermo-mechanical FE analysis., *Int. J. Num. Meth. Engng.*, **50**: 299–323, 2001.

[3] J. Ghaboussi, J.H. Garrett, X. Wu. Knowledge-Based Modelling of Material Behaviour with Neural Networks, *Journal of Engineering Mechanics*, **117**: 132–151, 1991.

[4] J. Hertz, A. Krogh, G.R. Palmer. Introduction to the theory of neural computation, *Lecture Notes Vol. I, Santa Fe Institute Studies in the sciences of Complexity*, Addison-Wesley, 1991.

[5] Y.H. Hu, J-N. Hwang, eds. *Handbook of Neural Network Signal Processing.* CRC PRESS, 2002.

[6] M. Lefik. Use of artificial neural network to define a non-linear effective constitutive law for a composite. *Proceedings of XIII Polish Conference on Computer Methods in Mechanics,* XIII Polish Conference on Computer Methods in Mechanics, 723–732, Poznań 1997.

[7] M. Lefik, B.A. Schrefler. One-dimensional model of cable-in-conduit superconductors under cyclic loading using artificial neural networks. *Fusion Engineering and Design*, Elsevier Science, **60** (2): 105–117, 2002.

[8] M. Lefik, B.A. Schrefler. Artificial Neural Network for parameter identifications for an elasto-plastic model of super-conducting cable under cyclic loading. *Computers & Structures*, **80** (22): 1699–1713, 2002.

[9] M. Lefik, B.A. Schrefler. Artificial neural network as an incremental non-linear constitutive model for a finite element code. *Computer Methods in Applied Mechanics and Engineering*, Elsevier **192** (28–30): 3265–3283, 2003.

[10] M. Lefik, M. Wojciechowski. Computational tools based on artificial neural networks for analysis of composite materials. *Proceedings of AI-METH 2003 – Methods of Artificial Intelligence*, T. Burczyński, W. Cholewa, W. Moczulski, AI-METH 2003, November 5–7, Gliwice, 2003.

[11] S. Osowski. *Neuron Networks in Algorithmic Approach,* WNT, Warszawa, 1996.

[12] E. Sanchez-Palencia, *Non-Homogeneous Media and Vibration Theory.* Springer V, Berlin, 1980.
[13] Z. Waszczyszyn. Neural networks in plasticity: some new results and prospects of applications. *European Congress on Computational Methods in Applied Sciences and Engineering ECCOMAS 2000,* on CD, 2000.
[14] H.O. Peitgen, H. Jurgens, D. Saupe, *Limits of chaos. Fractals.* PWN, Warszawa, 2002.
[15] E. Ott, *Chaos in dynamical systems.* Cambridge University Press (2nd edition), 2002.